# **Mapping Research Requirements to Software Tools**

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## Framing the problem

Computational Science Software ...

- ... development is not (well) funded
- ... complexity is constantly increasing
- ... applicability is ever increasing

"For every complex problem there is an answer that is clear, simple, and wrong." H.L. Mencken (1949)

Developing your own software to implement every feature you need is perhaps "clear, simple and wrong".

"Being able to quickly evaluate good (scientific) software is increasingly important." C. Simmons (Just Now!)

#### So ...

- "If I have seen further, it is by standing on the shoulders of giants."
   I. Newton (1676)
- How can we identify software giants?
  - By enumerating requirements for a healthy software ecosystem, I hope to help you eliminate some software immediately
  - By providing processes and tools to help increase confidence, I hope to help you identify "good" software

# By the end of this talk ... (Goals/Outline)

- By the end of this talk, you should have a strategy for identifying whether software is worth evaluating
- By the end of this talk, you should have a game plan for building confidence in that software
- By the end of this talk, you should have a strategy for identifying what needs to be done before writing your own software
- By the end of this talk, you should have a game plan for writing your own software
- And, by the end of this talk, you will have heard generally accepted recommendations on scientific software as well as my highly-opinionated ones.

## **Version Control**

Minimum Guidelines – Actually using version control is the first step

#### Ideal Usage

- Put everything under version control
- Consider putting parts of your home directory under VC
- Use a consistent project structure and naming convention
- Commit often and in logical chunks
- Write meaningful commit messages
- Do all file operations in the VCS
- Set up change notifications if working with multiple people

#### **Common Version Control Systems**

- CVS
- Subversion (SVN)
- Git

## **Build Systems**

#### **Minimum Guidelines**

- Any build system is a good start but it's more than an alias
- Bash script
- Makefile
- Host specific Makefiles

#### A proper build system

- Cross platform
- Supports dependencies
- Supports finding #include files and libraries
- Supports parallel builds

#### Common build tools are Make, autoconf/automake, cmake, SCons

## Types of Documentation

#### Minimal acceptable documentation

- ReadMe file
- Commented Source
- Bugs / TODO lists
- Changelog

#### **Full Blown Documentation**

- Model, Discretization and Code Verification Documentation
- Requirements Document
- Autogenerated Technical Documentation and code structure from commented source files
- End-User Document if applicable
- Issue Tracking

# Scientific vs. Other Computing

- Scientific Computing
  - has to be right (or at least quantifiably wrong)
  - has to be fast
  - doesn't have to be easy to use
  - doesn't have to be pretty
- Other Types of Computing
  - errors are tolerable
  - speed is negotiable
  - often has to be pretty or easy to use or both

## Trust No One ... not even yourself

## **Unit Testing**

- Tests individual units of source code
- A Unit is the smallest testable part of a code
- Each Unit test should be independent from others
- Best to write the Unit and the Unit test at the same time

### **Regression Testing**

- Examples applications, unit tests, benchmark problems
- Catches unintended consequences of revisions
- Design of a suite of regression tests is an art
- Consider Code and Feature Coverage
- Test early and Test often
- Consider automating tests



## Why Verify?

#### Reinhart and Kenneth S. Rogoff: Growth in a Time of Debt, 2010

- "Our main result is that whereas the link between growth and debt seems relatively weak at normal debt levels, median growth rates for countries with public debt over roughly 90 percent of GDP are about one percent lower than otherwise"
- Dataset: "inflation and GDP growth across varying levels of debt for 20 advanced countries over the period 1946 through 2009"
- Rogoff testified in front of the Senate Budget Committee

#### **Inform Decision Makers**

- France: 30 Billion Euros in Austerity Measures
- England: 11.5 Billion Pounds in Austerity Measures
- "Spain's national budget cuts of almost 14 percent and regional budget cuts of up to 10 percent in health and social services"

## **Verification Failure**

### Excel Error!

- "Instead of AVERAGE(L30:L49), AVERAGE(L30:L44) was used."
- When corrected, GDP/DEBT ratios above 90% had growth of 2.2%



## Surely those were isolated events, right?

#### Other examples

- JPM VaR model for synthetic credit portfolio muted volatility by factor of two (\$6B in losses)
- Mars Surveyor (metric versus english units)
- Sleipner A North Sea Oil Platform Collapse ("serious error in the finite element analysis")
- Therac-25 (overdosed more than two-dozen patients with gamma radiation)
- Patriot missile mistiming (only off by one third of a second!)

## Verification

#### Verification of Scientific Software

• Verification ensures that the outputs of a computation accurately reflect the solution of the mathematical models.

#### **Code Verification**

- Ensuring that the code used in the simulation correctly implements the intended numerical discretization of the model.
  - This concept is \*not\* unique to Scientific Software

#### Solution Verification

- Are the errors from the numerical discretization sufficiently small?
- Is the convergence rate consistent with the numerical scheme?

## Method of Exact Solutions

• Numerically solve the governing equations for which the solution can be determined analytically.

## MMS

- Often, analytical solutions either:
  - Do not exist (Navier-Stokes)
  - Do not fully exercise equations (e.g. a symmetric solution, nonlinearities)
- Alleviate this using Method of Manufactured Solutions (MMS)
  - Simply put, we "create" our own solutions

# Manufactured solution to Laplace's Equation

Laplace's Equation:

$$\nabla^2 \phi = 0$$

In two dimensions:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

"Manufacture" a solution, with two constants:

$$\phi(x,y) = (Ly - y)^2 (Ly + y)^2 + (Lx - x)^2 (Lx + x)^2$$

## Calculating the Source Term

We insert our manufactured solution back into the governing equations:

$$\frac{\partial^2 ((Lx-x)^2 (Lx+x)^2)}{\partial x^2} + \frac{\partial^2 ((Ly-y)^2 (Ly+y)^2)}{\partial y^2} = 0$$

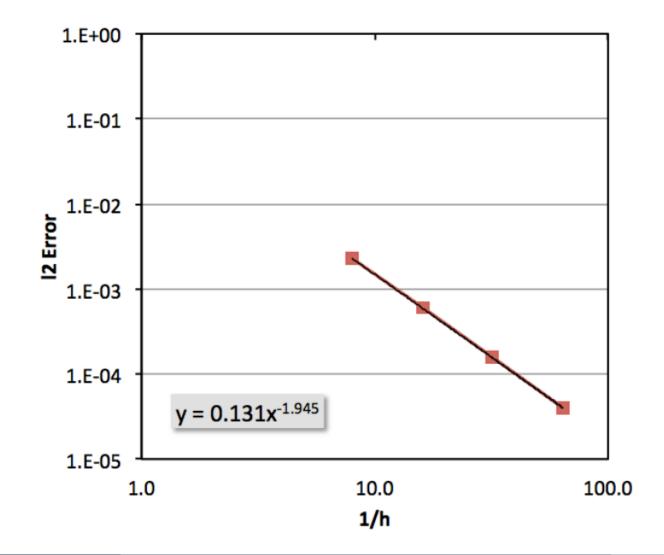
$$=2(Lx - x)^{2} - 8(Lx - x)(Lx + x) + 2(Lx + x)^{2}$$
$$+2(Ly - y)^{2} - 8(Ly - y)(Ly + y) + 2(Ly + y)^{2}$$
$$\neq 0$$

This does not satisfy Laplace's Equation!

To balance the equation, add the residual to the RHS as a source term.

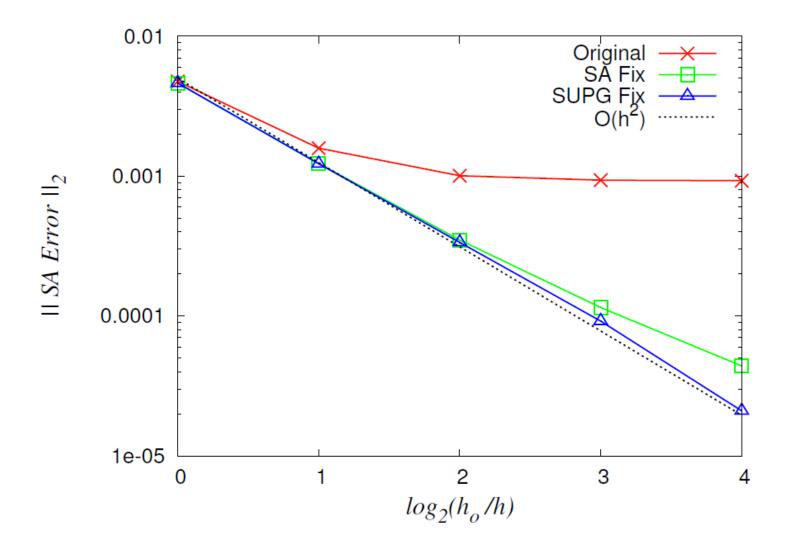
## Example Results: What we're hoping for

2nd Order Central Finite-difference Scheme



C. Simmons

## This Process Finds Bugs



C. Simmons

## Useful for Detecting Subtle Bugs

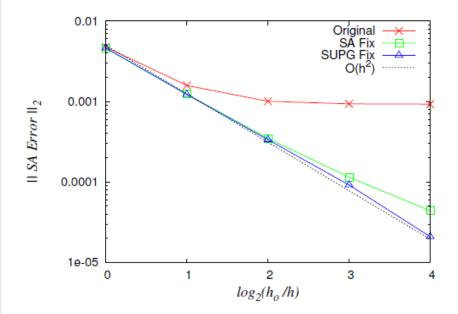
#### Verification of FIN-S

- FANS, Spalart-Allmaras
- Derivative:

$$\frac{d(sa)}{dx} = \frac{1}{\rho} * \left(\frac{d(\rho * sa)}{dx} - sa\frac{d\rho}{dx}\right)$$

• In code:

$$\frac{d(sa)}{dx} = \frac{1}{\rho} * \frac{d(\rho * sa)}{dx} - sa\frac{d\rho}{dx}$$





## What we have learned:

- We can generate MS even for systems that do not have solutions
- The MMS is a powerful method to verify rates of convergence

## Why is this not more commonly done?

- Solution Generation is complex and time intensive
- Providing high accuracy code is not as easy as you would think (hope?)

## A Real Example

#### **MMS Creation Process**

- Start by "manufacturing" a suitable closed-form exact solution
- For example, the 10 parameter trigonometric solution of the form: (Roy, 2002)

$$\hat{u}(x, y, z, t) = \hat{u}_0 + \hat{u}_x f_s \left(\frac{a_{\hat{u}x}\pi x}{L}\right) + \hat{u}_y f_s \left(\frac{a_{\hat{u}y}\pi y}{L}\right) + \\ + \hat{u}_z f_s \left(\frac{a_{\hat{u}z}\pi z}{L}\right) + \hat{u}_t f_s \left(\frac{a_{\hat{u}_t}\pi t}{L}\right)$$

 Apply this solution to equations of interest, solve for source terms (residual)

Accomplished using packages such as Maple, Mathematica, SymPy, Macsyma, etc.

## Maple MMS: 3D Navier-Stokes Energy Term

$$\begin{split} Qe &= -\frac{a_{\mu\pi}\pi p_{\pi}}{L} \frac{\gamma}{\gamma-1} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + u_{y} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{y}}{L} \frac{\gamma}{\gamma-1} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[u_{0} + v_{x} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right) + v_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + v_{z} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &- \frac{a_{\mu\pi}\pi p_{x}}{L} \frac{\gamma}{\gamma-1} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + w_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{x}}{L} \frac{\gamma}{\gamma-1} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + w_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{x}}{L} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + u_{y} \cos\left(\frac{a_{\mu\pi}\pi y}{L}\right) + u_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] \left[\left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + u_{y} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{x}}{2L} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + u_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] \left[\left[u_{0} + u_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + v_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{x}}{2L} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[v_{0} + v_{x} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right) + v_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + v_{z} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \frac{a_{\mu\pi}\pi p_{x}}{2L} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + u_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) \left[v_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + w_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi y}{L}\right) + w_{z} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{x} \cos\left(\frac{a_{\mu\pi}\pi x}{L}\right)\right] + \\ &+ \left[w_{0} + w_{x} \sin\left(\frac{a_{\mu\pi}\pi x}{L}\right) + w_{y} \sin\left(\frac{a_{\mu\pi}\pi x$$

### But wait, there's more!

$-a_{\mu\nu}^{2}\pi^{2}k\rho_{\tau}\left[2\cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)^{2}\rho_{x}+\sin\left(\frac{a_{\mu\nu}\pi x}{L}\right)\left[\rho_{1}+\rho_{\ell}\sin\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{3}\cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{2}\sin\left(\frac{a_{\mu\nu}\pi x}{L}\right)\right]\left[\rho_{0}+\rho_{\ell}\cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{3}\sin\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{5}\cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{5}\cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_{5}\sin\left(\frac{a_{\mu\nu}\pi x}{L}\right)+\rho_$	$\left(\frac{a\pi z}{L}\right)$
$= RL^2 \left[ \rho_{\theta} + \rho_{\theta} \sin \left( \frac{a_{\mu} \pi \pi}{L} \right) + \rho_{\theta} \cos \left( \frac{a_{\mu} \pi \pi}{L} \right) + \rho_{z} \sin \left( \frac{a_{\mu z} \pi z}{L} \right) \right]^d$	
$=a_{pq}^{2}\pi^{2}kp_{y}\left[2\sin\left(\frac{a_{pq}\pi y}{L}\right)^{2}\rho_{y}+\cos\left(\frac{a_{pq}\pi y}{L}\right)\left[\rho_{0}+\rho_{x}\sin\left(\frac{a_{pq}\pi x}{L}\right)+\rho_{y}\cos\left(\frac{a_{pq}\pi y}{L}\right)+\rho_{z}\sin\left(\frac{a_{pq}\pi y}{L}\right)\right]\left[\rho_{0}+\rho_{x}\cos\left(\frac{a_{pq}\pi y}{L}\right)+\rho_{y}\sin\left(\frac{a_{pq}\pi y}{L}\right)+\rho_{y}\sin\left(\frac{a_{pq}\pi y}{L}\right)+\rho_{y}\cos\left(\frac{a_{pq}\pi y}{L}\right)\right]\right]$	$\left[\frac{\pi z}{L}\right]$
$= RL^{2} \left[\rho_{h} + \rho_{s} \sin \left(\frac{a_{yx} \tau x}{L}\right) + \rho_{s} \cos \left(\frac{a_{yy} \tau y}{L}\right) + \rho_{s} \sin \left(\frac{a_{yx} \tau x}{L}\right)\right]^{3}$	
$-a_{\rho_{0}}^{2}\pi^{2}k\rho_{e}\left[2\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)^{4}\rho_{e}+\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)\left[\rho_{0}+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)\right]\left[p_{0}+p_{e}\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)\right]\left[p_{0}+p_{e}\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)\right]\left[p_{0}+p_{e}\cos\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)+\rho_{e}\sin\left(\frac{a_{\rho_{0}}\pi^{2}}{L}\right)$	( <sup>#2</sup> )]
$RL^2 \left[\rho_0 + \rho_x \sin\left(\frac{a_{\mu\mu}\pi x}{L}\right) + \rho_y \cos\left(\frac{a_{\mu\mu}\pi y}{L}\right) + \rho_s \sin\left(\frac{a_{\mu\mu}\pi z}{L}\right)\right]^3$	
$-\frac{4}{3}\frac{a_{xx}^2\pi^2\mu u_x}{L^2}\left[\cos\left(\frac{a_{xx}\pi x}{L}\right)^2 u_x - \sin\left(\frac{a_{xx}\pi x}{L}\right)\left[u_0 + u_x\sin\left(\frac{a_{xx}\pi x}{L}\right) + u_y\cos\left(\frac{a_{xy}\pi x}{L}\right) + u_z\cos\left(\frac{a_{xy}\pi x}{L}\right)\right] + u_y\cos\left(\frac{a_{xy}\pi x}{L}\right) + u_y\sin\left(\frac{a_{xy}\pi x}{L}\right) + u$	
$\frac{a_{uy}^2\pi^2\rho u_g}{L^2}\left[\sin\left(\frac{a_{uy}\pi y}{L}\right)^2 u_\eta - \cos\left(\frac{a_{uy}\pi y}{L}\right)\left[u_0 + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + u_\eta \cos\left(\frac{a_{uy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{uy}\pi x}{L}\right)\right]\right] + \frac{a_{uy}^2\pi^2\rho u_g}{L^2}\left[\sin\left(\frac{a_{uy}\pi x}{L}\right)^2 + \frac{a_{uy}^2\pi^2\rho u_g}{L^2}\right]$	
$-\frac{a_{as}^2\pi^2\mu u_{\pi}}{L^2}\left[\sin\left(\frac{a_{as}\pi z}{L}\right)^2 u_{\pi} - \cos\left(\frac{\phi_{as}\pi z}{L}\right)\left[u_0 + u_{\pi}\sin\left(\frac{a_{as}\pi z}{L}\right) + u_{\pi}\cos\left(\frac{a_{ag}\pi g}{L}\right) + u_{\pi}\cos\left(\frac{a_{ag}\pi g}{L}\right)\right] + u_{\pi}\cos\left(\frac{a_{ag}\pi g}{L}\right) + u_{\pi}\cos\left(\frac{a_{\pi}g}{L}\right) + u_{\pi}\cos\left(\frac{a_{\pi}g}{$	
$\frac{a_{xx}^2a_{xy}^2a_{yyz}^2}{L^2}\left[\sin\left(\frac{a_{xx}\pi L}{L}\right)^2v_x - \cos\left(\frac{a_{xx}\pi L}{L}\right)\left[v_0 + v_x\cos\left(\frac{a_{xy}\pi x}{L}\right) + v_y\sin\left(\frac{a_{xy}\pi x}{L}\right) + v_z\sin\left(\frac{a_{xy}\pi x}{L}\right)\right]\right] + \frac{a_{xy}^2a_{yyz}^2}{L^2}$	
$ -\frac{4}{3}\frac{a_g^2 y^2 \mu v_g}{L^2} \left[ \cos\left(\frac{a_{eg} \pi y}{L}\right)^2 v_g - \sin\left(\frac{a_{eg} \pi y}{L}\right) \left[ v_0 + v_x \cos\left(\frac{a_{eg} \pi x}{L}\right) + v_y \sin\left(\frac{a_{eg} \pi y}{L}\right) + v_z \sin\left(\frac{a_{eg} \pi y}{L}\right) \right] + v_y \sin\left(\frac{a_{eg} \pi y}{L}\right) \right] + v_y \sin\left(\frac{a_{eg} \pi y}{L}\right) \right] + v_y \sin\left(\frac{a_{eg} \pi y}{L}\right) + v_y \sin\left$	
$\frac{a_{xz}^2\pi^2\mu v_z}{L^2}\left[\cos\left(\frac{a_{xz}\pi z}{L}\right)^2 v_z - \sin\left(\frac{a_{xz}\pi z}{L}\right)\left[v_0 + v_x\cos\left(\frac{a_{xz}\pi z}{L}\right) + v_0\sin\left(\frac{a_{xz}\pi y}{L}\right) + v_z\sin\left(\frac{a_{xz}\pi z}{L}\right)\right]\right] + \frac{a_{xz}^2\pi^2\mu v_z}{L^2}\left[\cos\left(\frac{a_{xz}\pi z}{L}\right)^2 + \sin\left(\frac{a_{xz}\pi z}{L}\right)\right]$	
$\frac{a_{uu}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \left[ i \eta_1 + v_x \sin \left( \frac{a_{uv} \pi x}{L} \right) + w_y \sin \left( \frac{a_{uv} \pi y}{L} \right) + u_z \cos \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \cos \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right] + \frac{a_{uv}^2 \pi^2 \mu x_x}{L^2} \left[ \sin \left( \frac{a_{uv} \pi x}{L} \right)^2 w_x - \sin \left( \frac{a_{uv} \pi x}{L} \right) \right] \right]$	
$\frac{a_{\pi\pi}^2 \pi^2 \mu w_v}{L^2} \left[ \cos\left(\frac{a_{\pi\pi} \pi^2}{L}\right)^2 w_v - \sin\left(\frac{a_{\pi\pi} \pi^2}{L}\right) \left[ u_0 + w_s \sin\left(\frac{a_{\pi\pi} \pi x}{L}\right) + w_y \sin\left(\frac{a_{\pi\pi} \pi^2}{L}\right) + w_s \cos\left(\frac{a_{\pi\pi} \pi^2}{L}\right) \right] \right] + \frac{a_{\pi\pi}^2 \pi^2 \mu w_v}{L^2} \left[ \cos\left(\frac{a_{\pi\pi} \pi^2}{L}\right)^2 + \frac{a_{\pi\pi}^2 \pi^2}{L^2} + \frac{a_{\pi\pi\pi}^2 \pi^2}{L^2} + \frac{a_{\pi\pi\pi\pi}^2 \pi^2}{L^2} + \frac{a_{\pi\pi\pi\pi\pi}^2 \pi^2}{L^2} + \frac{a_{\pi\pi\pi\pi\pi\pi\pi\pi}^2 \pi^2}{L^2} + a_{\pi$	
$-\frac{4}{3}\frac{a_{zx}^2\pi^2\mu c_z^3}{L^2}\left[\sin\left(\frac{a_{zx}\pi z}{L}\right)^2w_z - \cos\left(\frac{a_{zx}\pi z}{L}\right)\left[w_0 + w_x\sin\left(\frac{a_{zx}\pi x}{L}\right) + w_x\sin\left(\frac{a_{zy}\pi x}{L}\right) + w_z\cos\left(\frac{a_{zy}\pi z}{L}\right)\right]\right] + \frac{4}{3}\frac{a_{zy}\pi^2}{L^2}\left[\sin\left(\frac{a_{zy}\pi z}{L}\right)^2w_z - \cos\left(\frac{a_{zy}\pi z}{L}\right)^2w_z\right] + \frac{4}{3}\frac{a_{zy}\pi^2}{L^2}\left[\sin\left(\frac{a_{zy}\pi z}{L}\right)^2w_z\right] + \frac{4}{3}\frac{a_{zy}\pi^2}{L^2}\left[\sin\left(\frac{a_{zy}\pi z}{L}\right)^2w_z\right] + \frac{4}{3}\frac{a_{zy}\pi z}{L^2}\left[\sin\left(\frac{a_{zy}\pi z}{L}\right)^2w_z\right] + \frac{4}{3}\frac{a_{zy}\pi z}{L^$	

$\left[\frac{a\pi z}{L}\right]$	$-\frac{a_{xx}\tau x_x}{L}\sin\left(\frac{a_{xx}\pi x}{L}\right)\left[u_0+u_x\sin\left(\frac{a_{xx}\pi x}{L}\right)+u_0\cos\left(\frac{a_{yx}\pi y}{L}\right)+u_x\cos\left(\frac{a_{yx}\pi y}{L}\right)\right]\left[\rho_0+\rho_x\sin\left(\frac{a_{yx}\pi x}{L}\right)+\rho_y\cos\left(\frac{a_{yy}\pi y}{L}\right)+\rho_z\sin\left(\frac{a_{yx}\pi x}{L}\right)\right]$
+	$\left[v_0 + v_s \cos\left(\frac{a_{os} \pi x}{L}\right) + v_s \sin\left(\frac{a_{os} \pi y}{L}\right) + v_s \sin\left(\frac{a_{os} \pi y}{L}\right)\right] +$
$\left[\frac{\pi z}{L}\right]$	$+\frac{a_{xy}\pi y}{2L}\cos\left(\frac{a_{xy}\pi y}{L}\right)\left\{\left(\left[u_{0}+u_{x}\sin\left(\frac{a_{xx}\pi x}{L}\right)+u_{y}\cos\left(\frac{a_{xy}\pi y}{L}\right)+u_{z}\cos\left(\frac{a_{xy}\pi y}{L}\right)\right]^{2}+\left[u_{0}+u_{y}\sin\left(\frac{a_{xy}\pi y}{L}\right)+u_{y}\sin\left(\frac{a_{xy}\pi y}{L}\right)+u_{z}\cos\left(\frac{a_{xy}\pi y}{L}\right)\right]^{2}+\frac{a_{xy}\pi y}{L}\right\}\right\}$
+	$+3\left[v_0+v_x\cos\left(\frac{a_{xy}\pi x}{L}\right)+v_y\sin\left(\frac{a_{xy}\pi x}{L}\right)+v_z\sin\left(\frac{a_{xy}\pi x}{L}\right)\right]^2\right)\left[\rho_0+\rho_x\sin\left(\frac{a_{yy}\pi x}{L}\right)+\rho_y\cos\left(\frac{a_{yy}\pi x}{L}\right)+\rho_z\sin\left(\frac{a_{yy}\pi x}{L}\right)\right]+\frac{1}{2}\left[v_0+v_x\cos\left(\frac{a_{xy}\pi x}{L}\right)+v_x\sin\left(\frac{a_{xy}\pi x}{L}\right)+v_x\sin\left(\frac{a_{xy}\pi x}{L}\right)\right]^2\right]$
[ <sup>πα</sup> ]	+ $\left[p_0 + p_s \cos\left(\frac{a_{\mu\nu}\pi x}{L}\right) + p_s \sin\left(\frac{a_{\mu\nu}\pi y}{L}\right) + p_s \cos\left(\frac{a_{\mu\nu}\pi x}{L}\right)\right]\frac{2\gamma}{\gamma - 1}\right\} +$
	$+\frac{a_{12}\pi v_z}{L} \mathrm{rns}\left(\frac{a_{12}\pi z}{L}\right) \left[w_0+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_y\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)\right] \left[\rho_0+\rho_s\sin\left(\frac{a_{22}\pi z}{L}\right)+\rho_0\cos\left(\frac{a_{22}\pi y}{L}\right)+\rho_s\sin\left(\frac{a_{22}\pi z}{L}\right)\right] + \frac{1}{2}\left[w_0+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)\right] + \frac{1}{2}\left[w_0+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)\right] + \frac{1}{2}\left[w_0+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}\pi z}{L}\right)+w_s\sin\left(\frac{a_{22}$
	+ $\left[v_0 + v_{\theta} \cos \left(\frac{a_{\theta,\eta} \pi x}{r}\right) + v_{\theta} \sin \left(\frac{a_{\theta,\eta} \pi y}{r}\right) + \pi_{\theta} \sin \left(\frac{a_{\theta,\eta} \pi x}{r}\right)\right] +$
	$+\frac{a_{uv}\pi w_{v}}{L}\cos\left(\frac{a_{uv}\pi \pi}{L}\right)\left[u_{0}+u_{s}\sin\left(\frac{a_{uv}\pi \pi}{L}\right)+u_{s}\cos\left(\frac{a_{uv}\pi y}{L}\right)+u_{s}\cos\left(\frac{a_{uv}\pi y}{L}\right)\right]\left[\rho_{0}+\rho_{s}\sin\left(\frac{a_{vv}\pi x}{L}\right)+\rho_{0}\cos\left(\frac{a_{vv}\pi y}{L}\right)+\rho_{s}\sin\left(\frac{a_{vv}\pi x}{L}\right)\right]-\frac{a_{uv}\pi x}{L}\sin\left(\frac{a_{vv}\pi x}{L}\right)\left[\frac{a_{uv}\pi x}{L}\right]$
	$-\left[u_0 + w_s \sin\left(\frac{\theta_{wc} \pi x}{T}\right) + w_y \sin\left(\frac{\theta_{ws} \pi y}{T}\right) + w_s \cos\left(\frac{\theta_{wz} \pi z}{T}\right)\right] +$
	$+\frac{a_{uy}\pi v_s}{t}\cos\left(\frac{a_{uy}\pi y}{t}\right)\left[v_0+v_s\cos\left(\frac{a_{ux}\pi x}{t}\right)+v_s\sin\left(\frac{a_{uy}\pi y}{t}\right)+v_s\sin\left(\frac{a_{ux}\pi z}{t}\right)\right]\left[\rho_0+\rho_s\sin\left(\frac{a_{ux}\pi x}{t}\right)+\rho_s\cos\left(\frac{a_{uy}\pi y}{t}\right)+\rho_s\sin\left(\frac{a_{ux}\pi z}{t}\right)\right]$
	$\left[u_0 + u_x \sin\left(\frac{w_{xx}\pi x}{L}\right) + u_y \sin\left(\frac{w_{xx}\pi y}{L}\right) + w_z \cos\left(\frac{w_{xx}\pi y}{L}\right)\right] + $
	$-\frac{a_{wz}\pi w_{0}}{2L}\sin\left(\frac{a_{wz}\pi z}{L}\right)\left\{\left(3\left u_{0}+u_{x}\sin\left(\frac{a_{wx}\pi z}{L}\right)+u_{y}\sin\left(\frac{a_{wy}\pi z}{L}\right)+u_{x}\cos\left(\frac{a_{wz}\pi z}{L}\right)\right)^{2}+\left[u_{1}+v_{x}\cos\left(\frac{a_{wz}\pi z}{L}\right)+v_{y}\sin\left(\frac{a_{wy}\pi z}{L}\right)+v_{z}\sin\left(\frac{a_{wz}\pi z}{L}\right)\right]^{2}+\frac{a_{wz}\pi z}{L}\sin\left(\frac{a_{wz}\pi z}{L}\right)+\frac{a_{wz}\pi z}{L$
	$+ \left[u_0 + u_x \sin\left(\frac{a_{yx}\pi x}{L}\right) + u_y \cos\left(\frac{a_{yy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{yx}\pi x}{L}\right)\right]^2\right) \left[\mu_0 + \mu_x \sin\left(\frac{a_{yx}\pi x}{L}\right) + \mu_y \cos\left(\frac{a_{yy}\pi y}{L}\right) + \mu_z \sin\left(\frac{a_{yz}\pi x}{L}\right)\right] + \frac{a_{yy}\pi x}{L} + a_$
	$+ \left[ p_0 + p_x \cos\left(\frac{a_{\mu\nu}\pi x}{L}\right) + p_y \sin\left(\frac{a_{\mu\mu}\pi y}{L}\right) + p_z \cos\left(\frac{a_{\mu\nu}\pi x}{L}\right) \right] \frac{2\gamma}{(\gamma - 1)} \right\} +$
	$u_{\mu\nu}^2 \pi^2 \dot{r} \mu_e \cos \left( \frac{\delta_{\mu\sigma} \pi T}{T} \right)$
	$+ \frac{L^2 R \left[\rho_3 + \rho_x \sin\left(\frac{a_{\mu\nu}\pi x}{L}\right) + \rho_8 \cos\left(\frac{a_{\mu\nu}\pi y}{L}\right) + \rho_x \sin\left(\frac{a_{\mu\nu}\pi z}{L}\right)\right]^+$
	$a_{pv}^2 \pi^2 k \rho_S \sin \left(\frac{\alpha_{ps} \pi_Y}{L}\right)$
	$+ \frac{1}{L^2 R} \left[ \rho_0 + \rho_e \sin\left(\frac{a_{\mu e} \pi x}{L}\right) + \rho_b \cos\left(\frac{a_{\mu q} \pi x}{L}\right) + \rho_c \sin\left(\frac{a_{\mu c} \pi x}{L}\right) \right]^+$
	$u_{xx}^2 \pi^3 k p_z \cos \left(\frac{u_x \pi}{L}\right)$
	$+ \frac{1}{L^2 R} \left[ \rho_0 + \rho_z \sin\left(\frac{a_{pc} \pi x}{L}\right) + \rho_y \cos\left(\frac{a_{pc} \pi p}{L}\right) + \rho_z \sin\left(\frac{a_{pc} \pi x}{L}\right) \right]^+$

		$\frac{\left(\frac{a_{\mu\sigma}\pi x}{L}\right)\sin\left(\frac{a_{\mu\sigma}\pi x}{L}\right)}{\left(\frac{a_{\mu\sigma}\pi y}{L}\right)}$ , $\left(\frac{a_{\mu\sigma}\pi y}{L}\right)$	2×12+
$L^{2}R _{Pa}$		$p \cos\left(\frac{a_{pq}\pi y}{L}\right) + p_1 \sin\left(\frac{a_{p_2}\pi}{L}\right)$	-11
		$\left(\frac{a_{\mu\nu}\pi y}{L}\right)\cos\left(\frac{a_{\mu\nu}\pi y}{L}\right)$	
$L^2R\left[\rho_0\right]$	$+ \rho_s \sin \left( \frac{s_{\rho \sigma} \pi x}{L} \right) + \rho_i$	$_{p} \cos \left(\frac{a_{\mu\nu}\pi y}{L}\right) + \rho_{s} \sin \left(\frac{a_{\mu s}\pi}{L}\right)$	=)] <sup>2</sup>
		$\left(\frac{a_{\mu\nu}\pi z}{L}\right) \sin\left(\frac{a_{\mu\nu}\pi z}{L}\right)$	
$L^2R\left[\rho_0\right]$	$+ \rho_T \sin\left(\frac{u_{\mu\nu}\pi x}{L}\right) + \rho_I$	$p \cos\left(\frac{a_{\rho_2}\pi y}{L}\right) + \rho_2 \sin\left(\frac{a_{\rho_2}\pi}{L}\right)$	···)]**
	$\frac{\pi^2 \mu u_x e_y}{D} \cos\left(\frac{a_{xx} \pi x}{L}\right)$		
$\frac{4}{3} \frac{a_{uu}a_{u}}{a_{uu}}$	$\frac{\pi^2 \mu m_s w_s}{I^3} \cos \left( \frac{w_{ss} \pi x}{L} \right)$	$\sin\left(\frac{a_{as}\pi z}{L}\right) +$	
$\frac{2a_{ag}a_{ab}}{l}$	$\frac{1}{2} \frac{r^2 \mu u_{\mu} v_{x}}{L} \sin\left(\frac{a_{xy} \pi y}{L}\right) s$	$\sin\left(\frac{a_{as}\pi x}{L}\right) +$	
	$\frac{\pi^2 \mu w_s w_s}{2} \cos \left( \frac{a_{sss} \pi x}{L} \right)$		
	$\frac{\pi^2 \mu v_Y w_z}{L^2} \cos\left(\frac{a_{vY} \pi y}{L}\right)$		
$2a_{zz}a_{xy}$	$\frac{\tau^2 \mu v_z u_y}{2} \cos\left(\frac{a_{zz} \tau z}{I}\right)$	$\left(\frac{a_{m}\pi y}{2}\right)$	

## C-code output

```
RHO = rho_0 + rho_x * sin(a_rhox * PI * x / L) + rho_y * cos(a_rhoy * PI * y / L) + rho_z * sin(a_rhoz * PI * z / L) + rho_t * sin(a_rhot * PI * t / L);
   U = u_0 + u_x * sin(a_ux * PI * x / L) + u_y * cos(a_uy * PI * y / L) + u_z * cos(a_uz * PI * z / L) + u_t * cos(a_ut * PI * t / L);
   V = v_0 + v_x * cos(a_vx * PI * x / L) + v_y * sin(a_vy * PI * y / L) + v_z * sin(a_vz * PI * z / L) + v_t * sin(a_vt * PI * t / L);
   W = w_0 + w_x * sin(a_wx * PI * x / L) + w_y * sin(a_wy * PI * y / L) + w_z * cos(a_wz * PI * z / L) + w_t * cos(a_wt * PI * t / L);
   P = p_0 + p_x * cos(a_px * PI * x / L) + p_y * sin(a_py * PI * y / L) + p_z * cos(a_pz * PI * z / L) + p_t * cos(a_pt * PI * t / L);
   M1 = A_mu * pow(P / R / RHD, 0.3e1 / 0.2e1);
   M2 = P / R / RHO + B_mu;
   MU - M1 / M2:
   Q_e = (0.2e1 * a_ux * u_x * cos(a_ux * PI * x / L) + a_vy * v_y * cos(a_vy * PI * y / L) - a_vz * v_z * sin(a_vz * PI * z / L)) * PI * RHO * U * U / L / 0.2e1 + (a_ux * u_x * cos(a_ux * PI
* x / L) + 0.3e1 * a vy * y * cos(a vy * PI * y / L) - a vz * v z * sin(a vz * PI * z / L)) * PI * RHD * V * V / L / 0.2e1 + (a ux * u x * cos(a ux * PI * x / L) + a vy * v v * cos(a vy *
PI * y / L) - 0.3e1 * a_wz * w_z * sin(a_wz * PI * z / L)) * PI * RHO * W * W / L / 0.2e1 + (0.4e1 * a_ux * a_ux * u_x * sin(a_ux * PI * x / L) + 0.3e1 * a_uy * a_uy * u_y * cos(a_uy * PI * y
/L) + 0.3e1 * a_uz * a_uz * a_uz * cos(a_uz * PI * z / L)) * MU * PI * PI * U * pov(L, -0.2e1) / 0.3e1 * a_vx * a_vx * v_x * cos(a_vx * PI * x / L) + 0.4e1 * a_vy * a_vy * v_y *
sin(a_vy * PI * y / L) + 0.3e1 * a_vz * a_vz * v_z * sin(a_vz * PI * z / L)) * MU * PI * PI * V * pov(L, -0.2e1) / 0.3e1 + (0.3e1 * a_vz * a_vz * v_z * sin(a_vz * PI * z / L) + 0.3e1 * a_vz *
awy * w y * sin(awy * PI * y / L) + 0.4ei * awz * awz * w z * cos(awz * PI * z / L)) * MU * PI * PI * W * pow(L, -0.2ei) / 0.3ei + (aux * u x * cos(aux * PI * x / L) + avy * v y *
cos(a_vy * PI * y / L) - a_vz * v_z * sin(a_vz * PI * z / L)) * PI * P / (Gamma - 0.1ei) / L - (0.2ei * a_rhox * a_px * rho_x * p_x * cos(a_rhox * PI * x / L) * sin(a_vx * PI * x / L) + 0.2ei
* a_rhoy * a_py * rho_y * p_y * sin(a_rhoy * PI * y / L) * cos(a_py * PI * y / L) + 0.2e1 * a_rhoz * a_pz * rho_z * p_z * cos(a_rhoz * PI * z / L) * sin(a_pz * PI * z / L)) * PI * PI * k *
pow(L, -0.2e1) / R * pow(RHD, -0.2e1) + (U * U + V * V + W * W) * a_rhot * PI * rho_t * cos(a_rhot * PI * t / L) / L / 0.2e1 - a_pt * PI * p_t * sin(a_pt * PI * t / L) / (Ganma - 0.1e1) / L -
(a_uy * u_y * sin(a_uy * PI * y / L) + a_vx * v_x * sin(a_vx * PI * x / L)) * PI * RHO * U * V / L - (a_ux * u_z * sin(a_ux * PI * z / L) - a_vx * v_x * cos(a_vx * PI * x / L)) * PI * RHO * U
* W / L + (a_vz * v_z * cos(a_vz * PI * z / L) + a_vv * v_v * cos(a_vv * PI * v / L)) * PI * RHO * V * W / L + (a_vz * a_vx * p_x * cos(a_vz * PI * x / L) + a_vv * a_vv * a_vv * p_v * sin(a_vv * PI
* y / L) + a_pz * a_pz * p_z * cos(a_pz * PI * z / L)) * PI * PI * k * pow(L, -0.2e1) / R / RHO - (0.4e1 * a_ux * a_ux * u_x * u_x * pow(cos(a_ux * PI * x / L), 0.2e1) - 0.4e1 * a_ux * a_vx *
u_x * v_y * cos(a_ux * PI * x / L) * cos(a_vy * PI * y / L) + 0.4e1 * a_ux * a_vz * u_x * v_z * cos(a_ux * PI * x / L) * sin(a_vz * PI * z / L) + 0.3e1 * a_uy * a_uy * u_y * u_y * a_vy * a_vy
pow(sin(a_uy * PI * y / L), 0.2e1) + 0.6e1 * a_uy * a_vx * u_y * v_x * sin(a_uy * PI * y / L) * sin(a_vx * PI * x / L) + 0.3e1 * a_uz * a_uz * u_z * u_z * pow(sin(a_uz * PI * z / L), 0.2e1) -
0.6e1 * a uz * a vz * u z * v x * sin(a uz * PI * z / L) * cos(a vz * PI * z / L) + 0.3e1 * a vz * a vz * v x * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v v * v x * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v v * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v x * v x * v x * pov(sin(a vz * PI * z / L), 0.2e1) + 0.4e1 * a vv * a vv * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * v * 
pow(cos(a_vy * PI * y / L), 0.2e1) + 0.4e1 * a_vy * a_vz * v_y * v_z * cos(a_vy * PI * y / L) * sin(a_vz * PI * z / L) + 0.3e1 * a_vz * a_vz * v_z * v_z * v_z * v_z * PI * z / L), 0.2e1) +
0.6e1 * a_vz * a_vy * v_z * v_y * cos(a_vz * PI * z / L) * cos(a_vy * PI * y / L) + 0.3e1 * a_vz * a_vz * u_z * u_z * u_z * v_c(cos(a_vz * PI * z / L), 0.2e1) + 0.3e1 * a_vy * u_y * u_y *
pow(cos(a wv * PI * v / L), 0.2ei) + 0.4ei * a wz * a wz * w z * w z * wz * pow(sin(a wz * PI * z / L), 0.2ei)) * MU * PI * PI * pow(L, -0.2ei) / 0.3ei + (a ux * u x * cos(a ux * PI * x / L) +
a_vy * v_y * cos(a_vy * PI * y / L) - a_vz * v_z * sin(a_vz * PI * z / L)) * PI * P / L - a_ut * PI * u_t * RHO * U * sin(a_ut * PI * t / L) / L + a_vt * PI * v_t * RHO * V * cos(a_vt * PI *
t / L) / L - a wt * PI * w t * RHO * W * sin(a wt * PI * t / L) / L + (U * U + V * V + W * W) * a rhox * PI * rho x * U * cos(a rhox * PI * x / L) / L / 0.2e1 - (U * U + V * V + W * W) *
a rhoy * PI * rho v * V * sin(a rhoy * PI * v / L) / L / 0.2e1 + (U * U + V * V + W * W) * a rhoz * PI * rho z * W * cos(a rhoz * PI * z / L) / L / 0.2e1 - (a rhoz * a rhoz * rho x * rho x *
sin(a_rhox * PI * x / L) + a_rhoy * a_rhoy * rho_y * cos(a_rhoy * PI * y / L) + a_rhoz * a_rhoz * rho_z * sin(a_rhoz * PI * z / L)) * PI * PI * k * P * pov(L, -0.2e1) / R * pov(RHO, -0.2e1) -
(0.2e1 * a_rhox * rho_x * rho_x * rho_x * rho_x * rho_x * pov(cos(a_rhox * PI * x / L), 0.2e1) + 0.2e1 * a_rhoy * a_rhoy * rho_y * rho_y * rho_y * pov(sin(a_rhoy * PI * y / L), 0.2e1) + 0.2e1 * a_rhox * a_rhox *
rho_z * rho_z * pow(cos(a_rhoz * PI * z / L), 0.2e1)) * PI * PI * k * P * pow(L, -0.2e1) / R * pow(RHO, -0.3e1) - (0.3e1 * a_rhox * a_uz * rho_x * u_z * cos(a_rhox * PI * x / L) * sin(a_uz *
PI * z / L) - 0.3e1 * a_rhox * a_wx * rho_x * v_x * cos(a_rhox * PI * x / L) * cos(a_wx * PI * x / L) + 0.3e1 * a_rhoy * a_vz * rho_y * v_z * sin(a_rhoy * PI * y / L) * cos(a_vz * PI * z / L)
+ 0.3e1 * a_rhoy * a_wy * rho_y * w_y * sin(a_rhoy * PI * y / L) * cos(a_wy * PI * y / L) + 0.2e1 * a_rhoz * a_ux * rho_z * u_x * cos(a_rhoz * PI * z / L) * cos(a_ux * PI * x / L) + 0.2e1 *
a_rhoz * a_vy * rho_z * v_y * cos(a_rhoz * PI * z / L) * cos(a_vy * PI * y / L) + 0.4e1 * a_rhoz * a_vz * rho_z * v_z * cos(a_rhoz * PI * z / L) * sin(a_vz * PI * z / L) * MU * (0.3e1 * B_mu
* R * RHO + P) * PI * PI * PI * PI * V / (B_mu * R * RHO + P) * pow(L, -0.2e1) / RHO / 0.6e1 - a_pz * PI * p_z * Gamma * W * sin(a_pz * PI * z / L) / (Gamma - 0.1e1) / L - (0.3e1 * a_pz * a_uy * p_z *
u_y * sin(a_px * PI * x / L) * sin(a_uy * PI * y / L) + 0.3e1 * a_px * a_vx * p_x * v_x * sin(a_px * PI * x / L) * sin(a_vx * PI * x / L) - 0.2e1 * a_py * a_ux * p_y * u_x * cos(a_py * PI * y
/L) * cos(a_ux * PI * x / L) + 0.4e1 * a_py * a_vy * p_y * v_y * cos(a_py * PI * y / L) * cos(a_vy * PI * y / L) + 0.2e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * sin(a_vz * PI *
z / L) - 0.3e1 * a_pz * a_vz * p_z * v_z * sin(a_pz * PI * z / L) * cos(a_vz * PI * z / L) - 0.3e1 * a_pz * a_vy * p_z * v_y * sin(a_pz * PI * z / L) * cos(a_vy * PI * y / L)) * (0.3e1 * B_mu
* R * RHD + P) * MJ * PI * PI * V / (B_nu * R * RHD + P) * pov(L, -0.2ei) / P / 0.6ei + a_py * PI * p_y * Ganna * V * cos(a_py * PI * y / L) / (Ganna - 0.1ei) / L - (0.3ei * a_rhox * a_uy *
rho_x * u_y * cos(a_rhox * PI * x / L) * sin(a_uy * PI * y / L) + 0.3e1 * a_rhox * a_vx * rho_x * v_x * cos(a_rhox * PI * x / L) * sin(a_vx * PI * x / L) - 0.2e1 * a_rhoy * a_ux * rho_y * u_x
* sin(a_rhoy * PI * y / L) * cos(a_ux * PI * x / L) + 0.4ei * a_rhoy * a_vy * rho_y * v_y * sin(a_rhoy * PI * y / L) * cos(a_vy * PI * y / L) + 0.2ei * a_rhoy * a_vz * rho_y * v_z *
sin(a_rhoy * PI * y / L) * sin(a_wz * PI * z / L) - 0.3e1 * a_rhoz * a_wz * rho_z * v_z * cos(a_rhoz * PI * z / L) * cos(a_wz * PI * z / L) - 0.3e1 * a_rhoz * a_wy * rho_z * w_y * cos(a_rhoz
* PI * z / L) * cos(a_wy * PI * y / L)) * MU * (0.3e1 * B_mu * R * RHO + P) * PI * V / (B_mu * R * RHO + P) * pow(L, -0.2e1) / RHO / 0.6e1 + (0.4e1 * a_rhox * a_wx * rho_x * u_x *
cos(a_rhox * PI * x / L) * cos(a_ux * PI * x / L) - 0.2e1 * a_rhox * a_vy * rho_x * v_y * cos(a_rhox * PI * x / L) * cos(a_vy * PI * y / L) + 0.2e1 * a_rhox * a_vz * rho_x * v_z * cos(a_rhox
* PI * x / L) * sin(a_wz * PI * z / L) + 0.3e1 * a_rhoy * a_wy * rho_y * u_y * sin(a_rhoy * PI * y / L) * sin(a_wy * PI * y / L) + 0.3e1 * a_rhoy * a_wx * rho_y * v_x * sin(a_rhoy * PI * y / L)
L) * sin(a_vx * PI * x / L) - 0.3e1 * a rhoz * a uz * rhoz * u z * cos(a rhoz * PI * z / L) * sin(a_uz * PI * z / L) + 0.3e1 * a rhoz * a vx * rhoz * u x * cos(a rhoz * PI * z / L) *
cos(a_wx * PI * x / L)) * MU * (0.3e1 * B_mu * R * RHO + P) * PI * U / (B_mu * R * RHO + P) * pow(L, -0.2e1) / RHO / 0.6e1 - a_px * PI * p_x * Ganma * U * sin(a_px * PI * x / L) / (Ganma
- 0.1e1) / L + (0.4e1 * a_px * a_ux * p_x * u_x * sin(a_px * PI * x / L) * cos(a_ux * PI * x / L) - 0.2e1 * a_px * a_vy * p_x * v_y * sin(a_px * PI * x / L) * cos(a_vy * PI * y / L) + 0.2e1 *
a_px * a_vz * p_x * v_z * sin(a_px * PI * x / L) * sin(a_vz * PI * z / L) + 0.3e1 * a_py * a_uy * p_y * u_y * cos(a_py * PI * y / L) * sin(a_uy * PI * y / L) + 0.3e1 * a_py * a_vx * p_y * v_x
* cos(a_py * PI * y / L) * sin(a_vx * PI * x / L) - 0.3e1 * a_pz * a_uz * p_z * u_z * sin(a_pz * PI * z / L) * sin(a_uz * PI * z / L) + 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * v_x * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * a_wx * p_z * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * a_wx * p_z * a_wx * p_z * sin(a_pz * PI * z / L) * 0.3e1 * a_pz * a_wx * p_z * a_wx * p_z
L) * cos(a_wx * PI * x / L)) * (0.3e1 * B_mu * R * RHD + P) * MU * PI * PI * U / (B_mu * R * RHD + P) * pow(L, -0.2e1) / P / 0.6e1 - (0.3e1 * a_px * a_uz * p_x * u_z * sin(a_px * PI * x / L)
* sin(a_uz * PI * z / L) - 0.3e1 * a_px * a_vx * p_x * u_x * sin(a_px * PI * x / L) * cos(a_vx * PI * x / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_py * PI * y / L) * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_py * a_vz * p_y * v_z * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * a_vz * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * cos(a_vx * PI * z / L) + 0.3e1 * a_vy * cos(a_vx * PI * z /
L) + 0.2e1 * a_py * a_wy * p_y * w_y * cos(a_py * PI * y / L) * cos(a_wy * PI * y / L) + 0.2e1 * a_pz * a_ux * p_z * u_x * sin(a_pz * PI * z / L) * cos(a_ux * PI * x / L) + 0.2e1 * a_pz *
a vv * p z * v v * sin(a pz * PI * z / L) * cos(a vv * PI * v / L) + 0.4e1 * a pz * a vz * p z * v z * sin(a pz * PI * z / L) * sin(a vz * PI * z / L)) * (0.3e1 * B nu * R * RHO + P) * MU *
PI * PI * W / (B_mu * R * RHO + P) * pow(L, -0.2e1) / P / 0.6e1;
```

## Manufactured Analytical Solutions Abstractions Library

Goal: Provide a repository and standardized interface for MMS usage

High Priority:

- Extreme fidelity to generated MMS
- Portability
- Traceability
- Extensible

## Low Priority:

• Speed/Performance

## Verifying the "Verifier"

Precision is not negotiable.

#### MASA Testing

- Error target < 1e-15
  - Absolute error on local machines
  - Relative error (other)
  - On all supported compiler sets
- -O0 not sufficient
  - -fp-model precise (Intel)
  - -fno-unsafe-math-optimizations (GNU)
  - -Kieee -Mnofpapprox (PGI)
- "make check"
  - Run by Buildbot every two hours

PASS:	init.sh
PASS:	misc
PASS:	fail_cond
PASS:	catch_exception
PASS:	register
PASS:	poly
PASS:	uninit
PASS:	vec
PASS:	purge
PASS:	heat_const_steady
PASS:	euler1d
:	
Final	izing MASA Tests

[nick@magus trunk]\$ make check

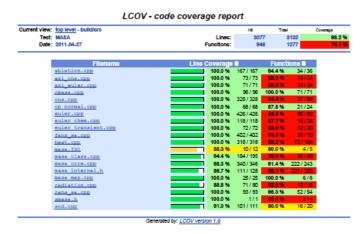
## Portability

#### Software Environment

- Built with: Autotools, C++
- Supports Intel, GNU, Portland Group compilers
- C/C++ interfaces
- Fortran interfaces provided through iso\_c\_bindings
  - Fortran 2003 Standard
- Python interfaces generated with SWIG

#### Testing

- GIT: version control
- Buildbot: automated testing
- GCOV: line coverage
  - ► 15,826 lines of code
  - 13,195 lines of testing
  - ► 98%+ line coverage



#### Traceability Doxygen provides code and model documentation

	Eul		

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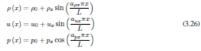
where  $\phi = \rho$ , u, v, w or p, and  $f_s(\cdot)$  functions denote either sine or cosine function. Note that in this case,  $\phi_x$ ,  $\phi_y$  and  $\phi_z$  are constants and the subscripts do not denote differentiation.

Atthough ? provide the constants used in the manufactured solutions for the 2D supersonic and subsonic cases for Euler and Navier-Stokes equations, only the source term for the 2D mass conservation equation (3.20) is presented.

Source terms for mass conservation  $(Q_{\mu})$ , momentum  $(Q_u, Q_u$  and  $Q_w)$  and total energy  $(Q_{e_i})$  equations are obtained by symbolic manipulations of compressible steady Euler equations above using Maple 13 (7) and are presented in the following sections for the one, two and three-dimensional cases.

#### 3.2.2.1 1D Steady Euler

The manufactured analytical solutions (3.52) for each one of the variables in one-dimensional case of Euler equations are:

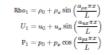


The MMS applied to Euler equations consists in modifying the 1D Euler equations (3.20) – (3.22) by adding a source term to the right-hand side of each equation:



so the modified set of equations (3.27) conveniently has the analytical solution given in Equation (3.53).

Source terms  $Q_{\mu}$ ,  $Q_{\mu}$  and  $Q_{e_{\mu}}$  are obtained by symbolic manipulations of equations above using Maple and are presented in the following sections. The following auxiliary variables have been included in order to improve readability and computational efficiency:



where the subscripts refer to the 1D case.

The mass conservation equation written as an operator is:

$$\mathcal{L} = \frac{\partial(\rho u)}{\partial x}$$

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3.2	Eni	ler	Eo	III SI	tions

k	u_0	u_x	u_y	u_z	v_0	L
v_0	V_X	v_y	v_z	w_0	W_X	w_y
rho_0	rho_x	rho_y	rho_z	p_0	p_y	p_z
a_px	a_py	a_pz	a_rhox	a_rhoy	a_rhoz	a_ux
a_uy	a_uz	a_vx	a_vy	a_vz	a_wx	a_wy
a_wz	mu	Gamma				

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#### Table 3.6: Parameters used by the 3D Steady Euler

masa\_eval\_2d\_exact\_u()

masa\_eval\_2d\_exact\_v()

masa\_eval\_2d\_exact\_p()

masa\_eval\_2d\_exact\_rho()

masa\_eval\_2d\_grad\_u()
masa\_eval\_2d\_grad\_v()

masa\_eval\_2d\_grad\_p()

masa\_eval\_2d\_grad\_rho()

#### 3.2.3.3 3D Steady Euler

Initialization:

euler\_3d

Functions:

masa\_init()

masa\_eval\_3d\_source\_rho\_u()

masa\_eval\_3d\_source\_rho\_v()

- · masa\_eval\_3d\_source\_rho\_w()
- masa\_eval\_3d\_source\_rho\_e()

masa\_eval\_3d\_source\_rho()

- masa\_eval\_3d\_exact\_u()
- masa\_eval\_3d\_exact\_v()
- masa\_eval\_3d\_exact\_w()
- masa\_eval\_3d\_exact\_p()

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## Available Solutions in MASA 0.50.0

Equations	Dimensions	Time
Euler	1,2,3, axi	Transient, Steady
Non linear heat conduction	1,2,3	Transient, Steady
Navier-Stokes	1,2,3, axi	Transient, Steady
N-S + Sutherland	3	Transient, Steady
N-S + ablation	1	Transient, Steady
Burgers	2	Transient, Steady
Sod Shock Tube	1	Transient
Euler + chemistry	1	Steady
RANS: Spalart-Allmaras	1	Steady
FANS: SA	2	Steady
FANS: SA + wall	2	Steady
Radiation	1	Steady
SMASA: Gaussian	1	Steady

## Importing New Solutions

#### Requirements

- Latex documents can be loaded directly into MASA documentation
  - Model document detailing analytical solution and source terms
  - Interface documentation detailing parameters and functions
- Source and analytical terms in C/C++/Fortran90
  - Can be integrated into your local MASA copy automagically (perl!)
  - Submit a patch
    - (unit tests would be nice)
- Willingness to share
- Publish these solutions!
- Success of MASA depends on use as a community tool

## Shortcomings

#### Symbolic Shortcomings

- Even with factorization, source terms still massive
- Generating manufactured solutions was a full time job at PECOS
- Everything we have discussed has been generated outside of MASA
  - Not thrilled with Maple, Mathematica

#### **Enter Automatic Differentiation**

- AD numerically evaluates the derivative of a function
  - applies chain rule repeatedly
- Superior error characteristics (round-off)
- Slow (but we barely care)
- Several libraries: NAG, Sacado, MetaPhysicL, etc.

## Snapshot

#### Release

- MASA 0.50.0 current release
- https://github.com/manufactured-solutions/MASA
- Open source, LGPL V2.1, free

#### **Publications**

- "MASA: a library for verification using manufactured analytical solutions"
  - DOI: 10.1007/s00366-012-0267-9
- A TRANSIENT MANUFACTURED SOLUTION FOR THE COMPRESSIBLE NAVIER-STOKES EQUATIONS WITH A POWER LAW VISCOSITY
- Manufactured Solutions for the Favre-Averaged Navier-Stokes Equations with Eddy-Viscosity Turbulence Models
- "A Linear Regression Framework for the Verification of Bayesian Model Calibration Algorithms" (submitted to ASME VVUQ)

## Researching

- Define what you want from all packages
- Define what a package aims to provide (scope)

## Reach out...

- Talk to other users in your field
- Talk to the developers of the package
  - Stable versus unstable code
  - How to contribute?
  - ► How to cite?
  - ► IRC / chat room
  - Mailing list

## Things to pay attention to...

- Variable names
- Portability
- Source formatting
- Default build system has warnings enabled
- Build system has a test flag

#### Other things to note

- Finding a bug doesn't rule it out
- Does the project accept patches?
- Licensing
- If TACC has a module for, it is worth evaluating
- Commercial software doesn't mean high quality
- Be polite to the developers

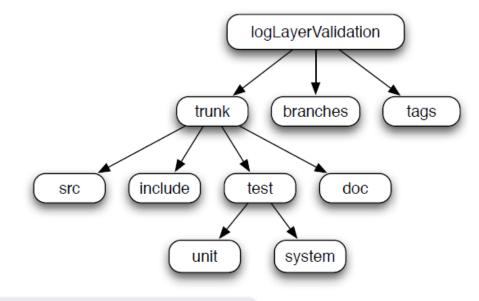
# Sometimes you have to bite the bullet ...

- Choose the right solution for your problem
  - Language
  - Hardware and portability
  - Open source / free vs performant (commercial) compilers / math libraries
- We ready to right code? Almost
  - Documentation
  - Requirements
  - Testing/Verification
  - Automation

# We need a place to work...

- To do any work, our project needs a place to live
- Should not just be in your home directory => use version control and work on a local copy
- E.g. : svn co https://collab.tacc.utexas.edu/...
- Start working...
  - Create directories: svn mkdir foo
  - Add files: svn add foo/bar.C
  - Commit to repo: svn ci -m "enlightening commit"

### Structure is important



- Trunk: primary development location
- Tags: "tagged" versions (e.g. releases)
- Braches: additional development locations for major refactoring

### Model Documentation

- Before you implement anything, you should be able to write down *precisely* what you plan to implement
- Easy to keep track of what you have done and are doing
- Helps to produce papers and thesis after code is done

### Model Documentation Cont.

- Ideally, everything necessary to implement your application, including:
  - All model equations, e.g., governing pdes or odes
  - Boundary conditions
  - Detailed discretization approach
  - Solution technique
  - Verification plan
- Implementation details are not required
  - That is, model document is not code documentation, not a developer guide, nor a user guide

# LogLayer Model Document

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### Select External Software

- Know what capability we desire (documented in model document)
- Now we need to decide how to implement that capability
- First question: What do I need to do myself, and what can I outsource?

### Select External Software

- First question: What do I need to do myself, and what can I outsource?
- Input parsing: GRVY, Boost program options
- Linear & Non-linear solvers: Petsc, Trilinos
- Finite Elements: Libmesh, Deal II
- General Scientific Comptuing: GSL
- UQ/Statistical Algorithms: QUESO, DAKOTA
- MANY, MANY, MANY MORE!

### Spring Mass Damper Example

- Interested in effects of uncertainty and validation philosophy
  - Input parser: GRVY
  - Numerical algorithms: GSL
  - Statistical algorithms: QUESO
- So, I only need to provide
  - Model routines
  - Data handling and uncertainty descriptions

## Choose build system

- Have selected some libraries to lighten the workload
- About ready to start coding
- Just need a way to compile & link reproducibly
- Make, Autotools, SCons: you make the call

### **Unit Testing**

- Ready to start writing code, which means we're ready to start writing tests!
- Main idea: Test a "unit" (e.g., a function) of code in isolation from everything else
- Motivation
  - Easier to write tests from small pieces of code rather than entire applications
  - Helps promote modular code design and frequent refactoring
  - Helps identify problem areas when system-level tests fail

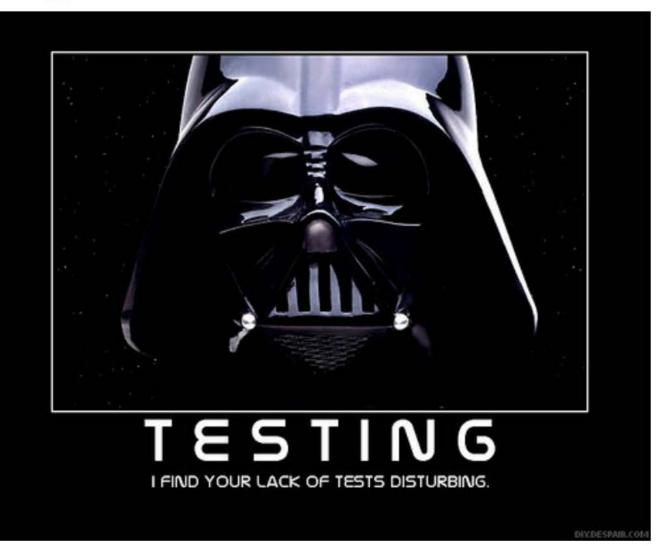
# **Unit Testing Tools**

- Requirements
  - A driver: something to simulate usage of the unit being tested
  - Data stubs: test data the the unit requires
  - Asserts: check that the unit behaves as expected
  - Automation: enable "continuous" testing

## **Unit Testing Tools**

- Many tools ("unit testing frameworks") available to help with these requirements:
- C/C++: CuTest, CppUnit, Boost Test, FCTX
- Fortran: fUnit, FRUIT
- MATLAB: MlUnit, MATLAB xUnit
- Python: nose
- http://en.wikipedia.org/wiki/List\_of\_unit\_testing\_frameworks

### Testing...



### Trust No One... not even yourself

- Unit Testing
  - Tests individual units of source code
  - A Unit is the smallest testable part of a code
  - Each Unit test should be independent from others
  - Write the unit and the test at the same time



## Trust No One... not even yourself

- Regression Testing
  - Example applications, unit tests, benchmark problems
  - Catches unintended consequences of revisions
  - Design of a suite of regressions tests is an art
  - Consider Code and Feature Coverage
  - Test early and test often
  - Automate testing

# Verification: Finding the Unknown Unknowns

- Solution Verification: Estimating Numerical Error
  - Discretization Error
  - Iterative Error
  - Round-off Error
- These errors cannot be avoided, but can (and must!) be quantified

# Verification: Finding the Unknown Unknowns

- Code Verification
  - Software bugs
  - Numerical algorithm weaknesses
  - Model implementation mistakes
- Codes cannot practically be proven errorfree, but can be proven to have errors. So we try our best to do the latter...

## Example: Hypersonic Flow App

#### Math, Software Components

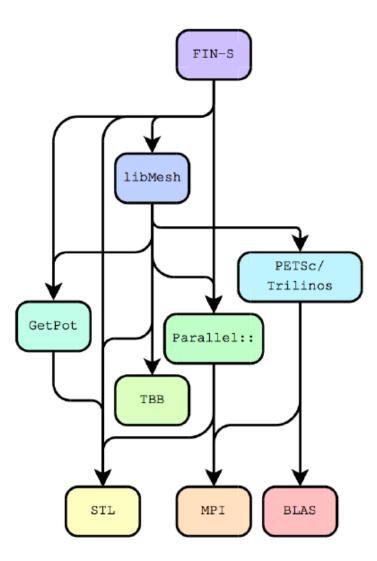
- Discretized Formulation
- Spatial Discretization
- Time Discretization
- System Assembly
- Nonlinear Solver
- Linear Solver
- I/O
- Postprocessing

Approximate LoC				
DPLR	46,000			
Coupling	6,000			
Ablation	6,000			
Radiation	6,000			
FIN-S	6,000			
libMesh	54,000			
PETSc	170,000			
Contrib	56,000			

### **Code Reuse**

#### Examples

- MPI, BLAS
- Aztec, MUMPS, PETSc
- deal.II, FEniCS, libMesh
- Don't reinvent the wheel unnecessarily!
- Time spent rewriting something old is time that could have been spent writing something new.
- More eyes == fewer bugs
- Extend existing capabilities where possible.



### **Unit Tests**

- Testing One Object At A Time
  - Reusable modules interact with all other code through a limited API
  - That API can be tested directly outside of application code
  - Test one method at a time, isolate problems locally
- Revise Software => Rerun Tests

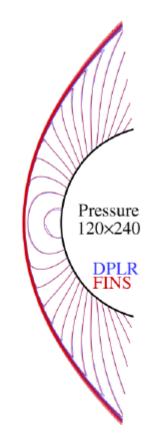
## Verification Benchmark Problems

- Choosing Test Problems
- Known solutions
  - Exact solution to discretized problem
  - Limit solution of continuous problem
  - Known quantities of interest
- Known asymptotic convergence rates
- Known residuals

# Code to Code Comparisons

#### "Lumped" Verification

- Differing results from:
  - Different models
  - Different formulations
  - Different discretizations



# **Hierarchic Models**

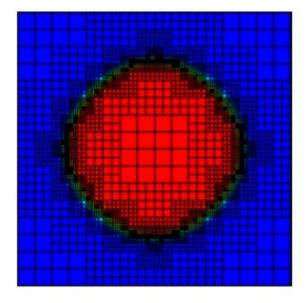
#### **Per-Operator Tesing**

- Coefficients selectively "turn off" parts of equations
- Allows easier construction of analytic solutions
- Assists with "narrowing down" other bugs

#### Model Simplification

- Model linearity can be tested in solver
- Reduce complex physics to simple physics case
- Code-to-code testing

# Symmetry Tests



### Symmetry In, Symmetry Out

- Mirror, radial symmetries
- Beware unstable solution modes!

# **Jacobian Verification**

### Jacobian Verification

#### Test Analytic vs. Numeric Jacobians

- Relative error in matrix norm
- If match isn't within tolerance, either:
  - The discretization or floating point error has overwhelmed the finite differenced Jacobian
    - Unlikely for good choices of finite difference perturbations
    - Can be investigated
  - The residual is non-differentiable at that iterate
    - Can be checked analytically
  - The Jacobian calculation is wrong
  - The residual calculation is wrong

### A Priori Asymptotic Convergence Rates

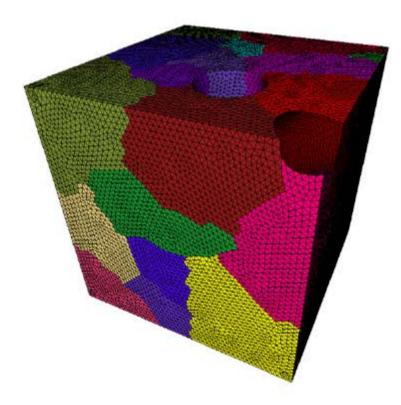
#### Pros

- Applicable to wide ranges of problems
- Does not require exact, or even manufactured, solution
- Part of solution verification, not just code verification

#### Cons

- Does not verify physics
- Requires asymptotic assumption
- Part of solution verification, not just code verification

# **Verification Never Ends**



#### **New Problems**

- New Optimizations
- New Features
- New Combinations
- New Physics
- New Data
- New Requirements

# A test suite that is never run is useless

- Automate testing
  - make check
  - svn hooks
  - crontab
  - Continuous Integration Server
- Standard procedure
  - Grab latest source code
  - Verify fresh configuration and compile
  - Verify solutions via regression tests
  - References solutions (and changelog) included in repository

### Example output

```
./rtest.sh
[...]
       _____
PECOS 1D Ablation Model: Version = 0.20
Build Date = 2009-05-02 11:59
Build Host = sqa.ices.utexas.edu
Build Arch = x86 64
Build Rev = r2650
Build Status = current
[...]
Verifying solution differences using a tolerance of: 1e-08
CN => Identical
       => Identical
0
       => Identical
CO
Recession => Identical
Temperature => Identical
NO
         => Identical
C3
  => Within tolerance (3.60731569E-19,3.62386249E-19)
Thank you, drive thru...
(rtest): PASSED: Test 1 (graphite/10species)
```

### Verification is iterative and ongoing

#### At least two possibilities:

- The observed phenomenon is real and must be understood
- The observed phenomenon is caused by a bug

### **Debugging Philosophy**

- Be skeptical of odd results—assume a bug and do everything to can to test for it
- View debugging as an opportunity to add new tests
- Once bug is found, add test that caught it

### The Bug and Updated Test Suite

#### **Broken Code**

```
for (unsigned int ii=1; ii<N; ++ii)
{
    const double yrat = p->expData->yPlus(ii)/p->expData->yPlus(ii-1);
    vp[ii] = modelRoutine<Mod>( *(p->physParams), yrat*zp, p->expData->deltaPlus() );
}
```

#### **Correct Code**

```
for (unsigned int ii=1; ii<N; ++ii)
{
    const double yrat = p->expData->yPlus(ii)/p->expData->yPlus(0);
    vp[ii] = modelRoutine<Mod>( *(p->physParams), yrat*zp, p->expData->deltaPlus() );
}
```

#### New Test

- Created unit test that checks output of function against saved output
- Not perfect, but better than nothing

### **Generally Accepted Recommendations**

#### **General Recommendations**

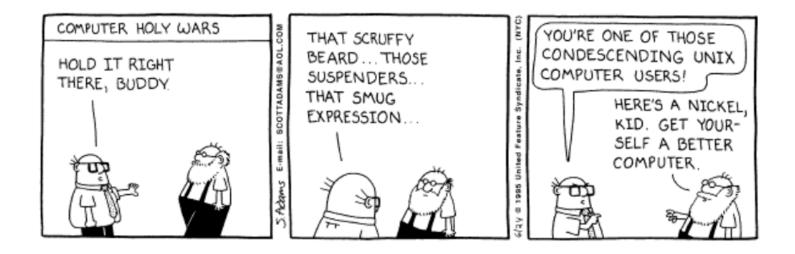
- Show other people your code early and often
- Reuse good software and evaluate multiple options
- Avoid platform dependent files (endianness)
- Always consider performance versus portability
- Prefer OSS solutions if possible
- Backup everything and anything

#### Don't reinvent the wheel

- Don't write your own input parser
- Don't write your own solver unless necessary
- Don't write anything until you evaluate options

Learn as much Unix as you can tolerate

### Unix: A Culture in Itself



"Two of the most famous products of Berkeley are LSD and Unix. I don't think that this is a coincidence." (Anonymous quote from The UNIX-HATERS Handbook.)

### Highly Opinionated Recommendations

#### Consistent tools from desktop to supercomputer

- Learn a console-based text edition (vi(m) / emacs)
- Learn a unix shell
- Learn LaTEX
- Learn a scripting language (python / perl)
- man (-k) is your friend

#### Understand:

- compile / link cycle
- static versus dynamic libraries
- processor optimization
- storage hierarchy (local versus network disk)
- environment variables (\$PATH and \$LD\_LIBRARY\_PATH)

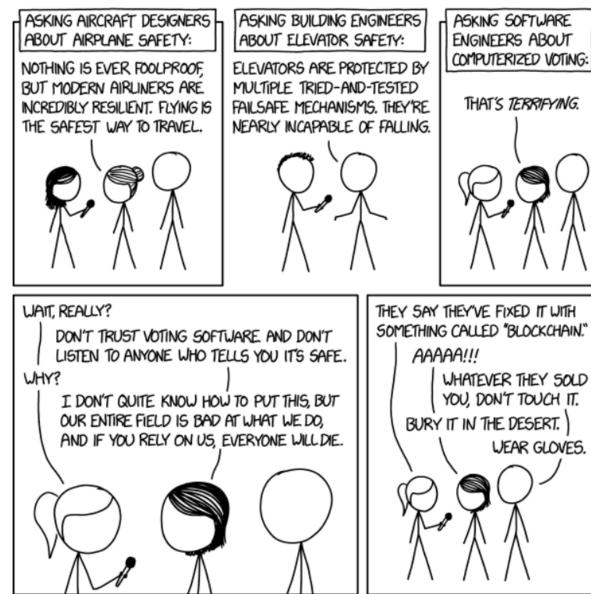
### More Highly Opinionated Recommendations

### More things to understand

- Actually learn how to use the compiler
- Learn about the hardware (memory, interconnect, disk)
- Learn about debugging and profiling
- Always have a restart capability

- Soapbox
  - Use text-based formats and version control whenever possible
  - Keep as much of your life as possible out of email
  - Check out OpenHPC!; Be NICE to SysAdmins

### Relevant XKCD



#### VOTE EARLY, VOTE OFTEN —

# Frozen machines, 243-percent turnout, and other woes in Georgia voting

A federal lawsuit alleges significant problems with voting in the Peach state.

JONATHAN M. GITLIN - 8/8/2018, 11:42 AM



**John McAfee** ♥
@officialmcafee

For all you naysayers who claim that "nothing is unhackable" & who don't believe that my Bitfi wallet is truly the world's first unhackable device, a \$100,000 bounty goes to anyone who can hack it. Money talks, bullshit walks. Details on Bitfi.com

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