Supercomputing in Plain English

Part III: Instruction Level Parallelism

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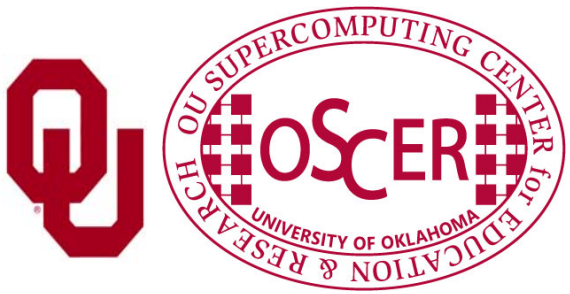
Tuesday February 17 2009



This is an experiment!

It's the nature of these kinds of videoconferences that **FAILURES ARE GUARANTEED TO HAPPEN! NO PROMISES!**

So, please bear with us. Hopefully everything will work out well enough.

If you lose your connection, you can retry the same kind of connection, or try connecting another way.

Remember, if all else fails, you always have the toll free phone bridge to fall back on.

Access Grid

This week's Access Grid (AG) venue: Monte Carlo.
If you aren't sure whether you have AG, you probably don't.

Tue Feb 17	Monte Carlo
Tue Feb 27	Helium
Tue March 3	Titan
Tue March 10	**NO WORKSHOP**
Tue March 17	**NO WORKSHOP**
Tue March 24	Axon
Tue March 31	Cactus
Tue Apr 7	Walkabout
Tue Apr 14	Cactus
Tue Apr 21	Verlet

Many thanks to John Chapman of U Arkansas for setting these up for us.

H.323 (Polycom etc)

If you want to use H.323 videoconferencing – for example, Polycom – then dial

`69.77.7.203##12345`

any time after 2:00pm. Please connect early, at least today.

For assistance, contact Andy Fleming of KanREN/Kan-ed (afleming@kanren.net or 785-865-6434).

KanREN/Kan-ed's H.323 system can handle up to 40 simultaneous H.323 connections. If you cannot connect, it may be that all 40 are already in use.

Many thanks to Andy and KanREN/Kan-ed for providing H.323 access.

iLinc

We have unlimited simultaneous iLinc connections available.

If you're already on the SiPE e-mail list, then you should receive an e-mail about iLinc before each session begins.

If you want to use iLinc, please follow the directions in the iLinc e-mail.

For iLinc, you **MUST** use either Windows (XP strongly preferred) or MacOS X with Internet Explorer.

To use iLinc, you'll need to download a client program to your PC. It's free, and setup should take only a few minutes.

Many thanks to Katherine Kantardjieff of California State U Fullerton for providing the iLinc licenses.

QuickTime Broadcaster

If you cannot connect via the Access Grid, H.323 or iLinc, then you can connect via QuickTime:

`rtsp://129.15.254.141/test_hpc09.sdp`

We recommend using QuickTime Player for this, because we've tested it successfully.

We recommend upgrading to the latest version at:

`http://www.apple.com/quicktime/`

When you run QuickTime Player, traverse the menus

File -> Open URL

Then paste in the rstp URL into the textbox, and click OK.

Many thanks to Kevin Blake of OU for setting up QuickTime Broadcaster for us.

Phone Bridge

If all else fails, you can call into our toll free phone bridge:

1-866-285-7778, access code 6483137#

Please mute yourself and use the phone to listen.

Don't worry, we'll call out slide numbers as we go.

Please use the phone bridge **ONLY** if you cannot connect any other way: the phone bridge is charged per connection per minute, so our preference is to minimize the number of connections.

Many thanks to Amy Apon and U Arkansas for providing the toll free phone bridge.

Please Mute Yourself

No matter how you connect, please mute yourself, so that we cannot hear you.

At OU, we will turn off the sound on all conferencing technologies.

That way, we won't have problems with echo cancellation.

Of course, that means we cannot hear questions.

So for questions, you'll need to send some kind of text.

Also, if you're on iLinc: **SIT ON YOUR HANDS!**

Please DON'T touch ANYTHING!

Questions via Text: iLinc or E-mail

Ask questions via text, using one of the following:

- iLinc's text messaging facility;
- e-mail to **sipe2009@gmail.com**.

All questions will be read out loud and then answered out loud.

Thanks for helping!

- OSCER operations staff (Brandon George, Dave Akin, Brett Zimmerman, Josh Alexander)
- OU Research Campus staff (Patrick Calhoun, Josh Maxey)
- Kevin Blake, OU IT (videographer)
- Katherine Kantardjieff, CSU Fullerton
- John Chapman and Amy Apon, U Arkansas
- Andy Fleming, KanREN/Kan-ed
- This material is based upon work supported by the National Science Foundation under Grant No. OCI-0636427, “CI-TEAM Demonstration: Cyberinfrastructure Education for Bioinformatics and Beyond.”

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Supercomputing Exercises

Want to do the "Supercomputing in Plain English" exercises?

- The first two exercises are already posted at:

  http://www.oscer.ou.edu/education.php

- If you don't yet have a supercomputer account, you can get a temporary account, just for the "Supercomputing in Plain English" exercises, by sending e-mail to:

  hneeman@ou.edu

Please note that this account is for doing the **exercises only**, and will be shut down at the end of the series.

- **This week's Arithmetic Operations exercise will give you experience benchmarking various arithmetic operations under various conditions.**

OK Supercomputing Symposium

Wed Oct 7 2009 @ OU

2003 Keynote:
Peter Freeman
NSF
Computer & Information Science & Engineering
Assistant Director

2004 Keynote:
Sangtae Kim
NSF Shared Cyberinfrastructure Division Director

2005 Keynote:
Walt Brooks
NASA Advanced Supercomputing Division Director

2006 Keynote:
Dan Atkins
Head of NSF's Office of Cyber-infrastructure

2007 Keynote:
Jay Boisseau
Director
Texas Advanced Computing Center
U. Texas Austin

2008 Keynote:
José Munoz
Deputy Office Director/ Senior Scientific Advisor
Office of Cyber-infrastructure
National Science Foundation

Parallel Programming Workshop
FREE! Tue Oct 6 2009 @ OU
Sponsored by SC09 Education Program
FREE! Symposium Wed Oct 7 2009 @ OU
http://symposium2009.oscer.ou.edu/

SC09 Summer Workshops

This coming summer, the SC09 Education Program, part of the SC09 (Supercomputing 2009) conference, is planning to hold two weeklong supercomputing-related workshops in Oklahoma, for **FREE** (except you pay your own travel):

- **At OU**: Parallel Programming & Cluster Computing, date to be decided, weeklong, for **FREE**
- **At OSU**: Computational Chemistry (tentative), date to be decided, weeklong, for **FREE**

We'll alert everyone when the details have been ironed out and the registration webpage opens.

Please note that you must apply for a seat, and acceptance **CANNOT** be guaranteed.

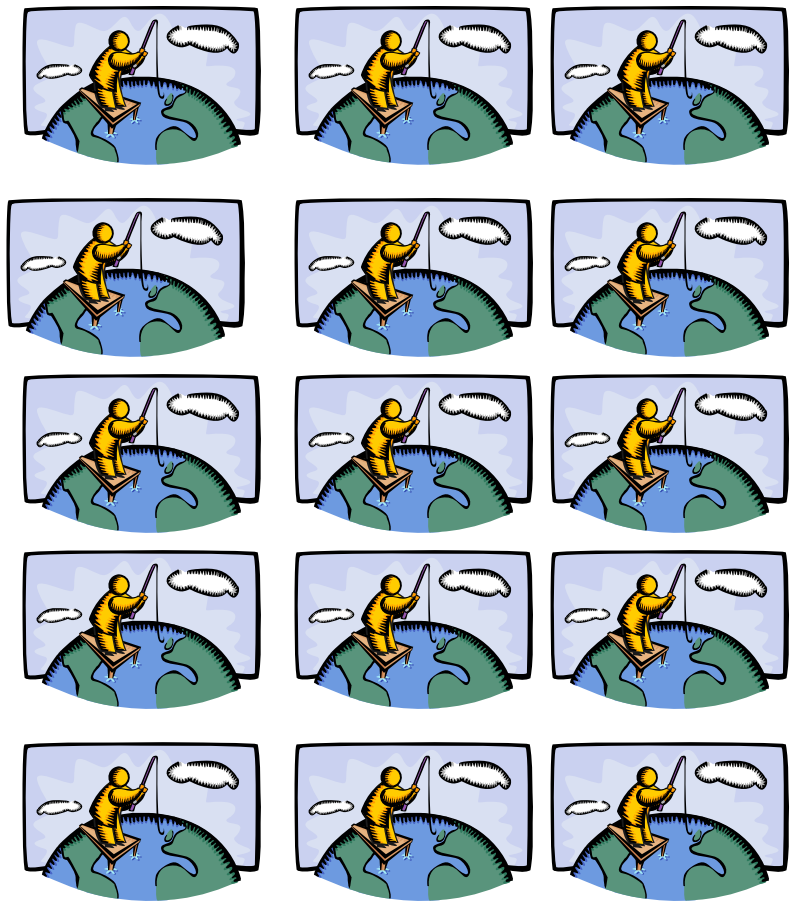
Outline

- What is Instruction-Level Parallelism?
- Scalar Operation
- Loops
- Pipelining
- Loop Performance
- Superpipelining
- Vectors
- A Real Example

Parallelism

Parallelism means doing multiple things at the same time: You can get more work done in the same time.

Less fish …



More fish!

What Is ILP?

Instruction-Level Parallelism (ILP) is a set of techniques for **executing multiple instructions at the same time within the same CPU core.**

(Note that ILP has nothing to do with multicore.)

The problem: The CPU has lots of circuitry, and at any given time, most of it is idle, which is wasteful.

The solution: Have different parts of the CPU work on different operations at the same time: If the CPU has the ability to work on 10 operations at a time, then the program can, in principle, run as much as 10 times as fast (although in practice, not quite so much).

DON'T PANIC!

Why You Shouldn't Panic

In general, the compiler and the CPU will do most of the heavy lifting for instruction-level parallelism.

BUT:

You need to be aware of ILP, because how your code is structured affects how much ILP the compiler and the CPU can give you.

Kinds of ILP

- ***Superscalar***: Perform multiple operations at the same time (for example, simultaneously perform an add, a multiply and a load).
- ***Pipeline***: Start performing an operation on one piece of data while finishing the same operation on another piece of data – perform different ***stages*** of the same operation on different sets of operands at the same time (like an assembly line).
- ***Superpipeline***: A combination of superscalar and pipelining – perform multiple pipelined operations at the same time.
- ***Vector***: Load multiple pieces of data into special registers and perform the same operation on all of them at the same time.

What's an Instruction?

- ***Memory***: For example, load a value from a specific address in main memory into a specific register, or store a value from a specific register into a specific address in main memory.
- ***Arithmetic***: For example, add two specific registers together and put their sum in a specific register – or subtract, multiply, divide, square root, etc.
- ***Logical***: For example, determine whether two registers both contain nonzero values ("**AND**").
- ***Branch***: Jump from one sequence of instructions to another (for example, function call).
- … and so on ….

What's a Cycle?

You've heard people talk about having a 2 GHz processor or a 3 GHz processor or whatever. (For example, Henry's laptop has a 1.83 GHz Pentium4 Centrino Duo.)

Inside every CPU is a little clock that ticks with a fixed frequency. We call each tick of the CPU clock a ***clock cycle*** or a ***cycle***.

So a 2 GHz processor has 2 billion clock cycles per second.

Typically, a primitive operation (for example, add, multiply, divide) takes a fixed number of cycles to execute (assuming no pipelining).

What's the Relevance of Cycles?

Typically, a primitive operation (for example, add, multiply, divide) takes a fixed number of cycles to execute (assuming no pipelining).

- IBM POWER4 [1]
  - Multiply or add: 6 cycles (64 bit floating point)
  - Load: 4 cycles from L1 cache
    14 cycles from L2 cache

- Intel Pentium4 EM64T (Core) [2]
  - Multiply: 7 cycles (64 bit floating point)
  - Add, subtract: 5 cycles (64 bit floating point)
  - Divide: 38 cycles (64 bit floating point)
  - Square root: 39 cycles (64 bit floating point)
  - Tangent: 240-300 cycles (64 bit floating point)

Scalar Operation

DON'T PANIC!

Scalar Operation

```
z = a * b + c * d;
```

How would this statement be executed?

1. Load **a** into register **R0**
2. Load **b** into **R1**
3. Multiply **R2** = **R0** * **R1**
4. Load **c** into **R3**
5. Load **d** into **R4**
6. Multiply **R5** = **R3** * **R4**
7. Add **R6** = **R2** + **R5**
8. Store **R6** into **z**

Does Order Matter?

```
z = a * b + c * d;
```

1. Load **a** into **R0**	1. Load **d** into **R0**
2. Load **b** into **R1**	2. Load **c** into **R1**
3. Multiply **R2 = R0 * R1**	3. Multiply **R2 = R0 * R1**
4. Load **c** into **R3**	4. Load **b** into **R3**
5. Load **d** into **R4**	5. Load **a** into **R4**
6. Multiply **R5 = R3 * R4**	6. Multiply **R5 = R3 * R4**
7. Add **R6 = R2 + R5**	7. Add **R6 = R2 + R5**
8. Store **R6** into **z**	8. Store **R6** into **z**

In the cases where order doesn't matter, we say that the operations are ***independent*** of one another.

Superscalar Operation

```
z = a * b + c * d;
```

1. Load **a** into **R0** **AND**
   load **b** into **R1**
2. Multiply **R2** = **R0** * **R1** **AND**
   load **c** into **R3** **AND**
   load **d** into **R4**
3. Multiply **R5** = **R3** * **R4**
4. Add **R6** = **R2** + **R5**
5. Store **R6** into **z**

If order doesn't matter,
then things can happen **simultaneously**.
So, we go from 8 operations down to 5.
(Note: there are lots of simplifying assumptions here.)

Loops

Loops Are Good

Most compilers are very good at optimizing **loops**, and not very good at optimizing other constructs.

Why?

```
DO index = 1, length
    dst(index) = src1(index) + src2(index)
END DO

for (index = 0; index < length; index++) {
    dst[index] = src1[index] + src2[index];
}
```

Why Loops Are Good

- Loops are **very common** in many programs.
- Also, it's easier to optimize loops than more arbitrary sequences of instructions: when a program does **the same thing over and over**, it's **easier to predict** what's likely to happen next.

So, hardware vendors have designed their products to be able to execute loops quickly.

DON'T PANIC!

Superscalar Loops

```
DO i = 1, length
   z(i) = a(i) * b(i) + c(i) * d(i)
END DO
```

Each of the iterations is **completely independent** of all of the other iterations; for example,

```
z(1) = a(1) * b(1) + c(1) * d(1)
```

has nothing to do with

```
z(2) = a(2) * b(2) + c(2) * d(2)
```

Operations that are independent of each other can be performed in **parallel**.

Superscalar Loops

```
for (i = 0; i < length; i++) {
   z[i] = a[i] * b[i] + c[i] * d[i];
}
```

1. Load **a[i]** into **R0** **AND** load **b[i]** into **R1**
2. Multiply **R2 = R0 * R1** **AND** load **c[i]** into **R3** **AND** load **d[i]** into **R4**
3. Multiply **R5 = R3 * R4** **AND** load **a[i+1]** into **R0** **AND** load **b[i+1]** into **R1**
4. Add **R6 = R2 + R5** **AND** load **c[i+1]** into **R3** **AND** load **d[i+1]** into **R4**
5. Store **R6** into **z[i]** **AND** multiply **R2 = R0 * R1**
6. etc etc etc

Once this loop is "in flight," each iteration adds only 2 operations to the total, not 8.

Example: IBM POWER4

8-way Superscalar: can execute up to 8 operations at the same time[1]

- 2 integer arithmetic or logical operations, and
- 2 floating point arithmetic operations, and
- 2 memory access (load or store) operations, and
- 1 branch operation, and
- 1 conditional operation

Pipelining

Pipelining

Pipelining is like an assembly line or a bucket brigade.

- An operation consists of multiple stages.
- After a particular set of operands

```
z(i) = a(i) * b(i) + c(i) * d(i)
```

completes a particular stage, they move into the next stage.

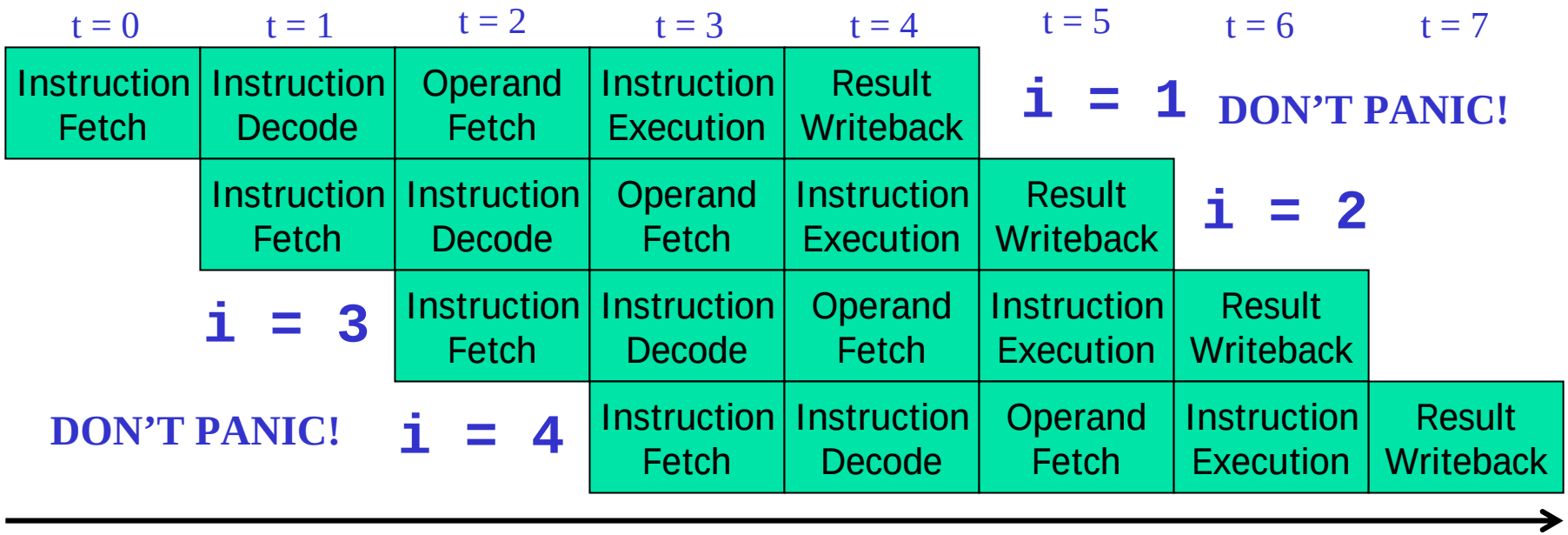
- Then, another set of operands

```
z(i+1) = a(i+1) * b(i+1) + c(i+1) * d(i+1)
```

can move into the stage that was just abandoned by the previous set.

DON'T PANIC!

Pipelining Example



If each stage takes, say, one CPU cycle, then once the loop gets going, each iteration of the loop increases the total time by only one cycle. So a loop of length 1000 takes only 1004 cycles. [3]

Pipelines: Example

- IBM POWER4: pipeline length ≅ 15 stages [1]

Some Simple Loops (F90)

```
DO index = 1, length
  dst(index) = src1(index) + src2(index)
END DO

DO index = 1, length
  dst(index) = src1(index) - src2(index)
END DO

DO index = 1, length
  dst(index) = src1(index) * src2(index)
END DO

DO index = 1, length
  dst(index) = src1(index) / src2(index)
END DO

DO index = 1, length
  sum = sum + src(index)
END DO
```

Reduction: convert array to scalar

Some Simple Loops (C)

```
for (index = 0; index < length; index++) {
  dst[index] = src1[index] + src2[index];
}

for (index = 0; index < length; index++) {
  dst[index] = src1[index] - src2[index];
}

for (index = 0; index < length; index++) {
  dst[index] = src1[index] * src2[index];
}

for (index = 0; index < length; index++) {
  dst[index] = src1[index] / src2[index];
}

for (index = 0; index < length; index++) {
  sum = sum + src[index];
}
```

Slightly Less Simple Loops (F90)

```
DO index = 1, length
  dst(index) = src1(index) ** src2(index) !! src1 ^ src2
END DO

DO index = 1, length
  dst(index) = MOD(src1(index), src2(index))
END DO

DO index = 1, length
  dst(index) = SQRT(src(index))
END DO

DO index = 1, length
  dst(index) = COS(src(index))
END DO

DO index = 1, length
  dst(index) = EXP(src(index))
END DO

DO index = 1, length
  dst(index) = LOG(src(index))
END DO
```



Slightly Less Simple Loops (C)

```
for (index = 0; index < length; index++) {
  dst[index] = pow(src1[index], src2[index]);
}

for (index = 0; index < length; index++) {
  dst[index] = src1[index] % src2[index];
}

for (index = 0; index < length; index++) {
  dst[index] = sqrt(src[index]);
}

for (index = 0; index < length; index++) {
  dst[index] = cos(src[index]);
}

for (index = 0; index < length; index++) {
  dst[index] = exp(src[index]);
}

for (index = 0; index < length; index++) {
  dst[index] = log(src[index]);
}
```

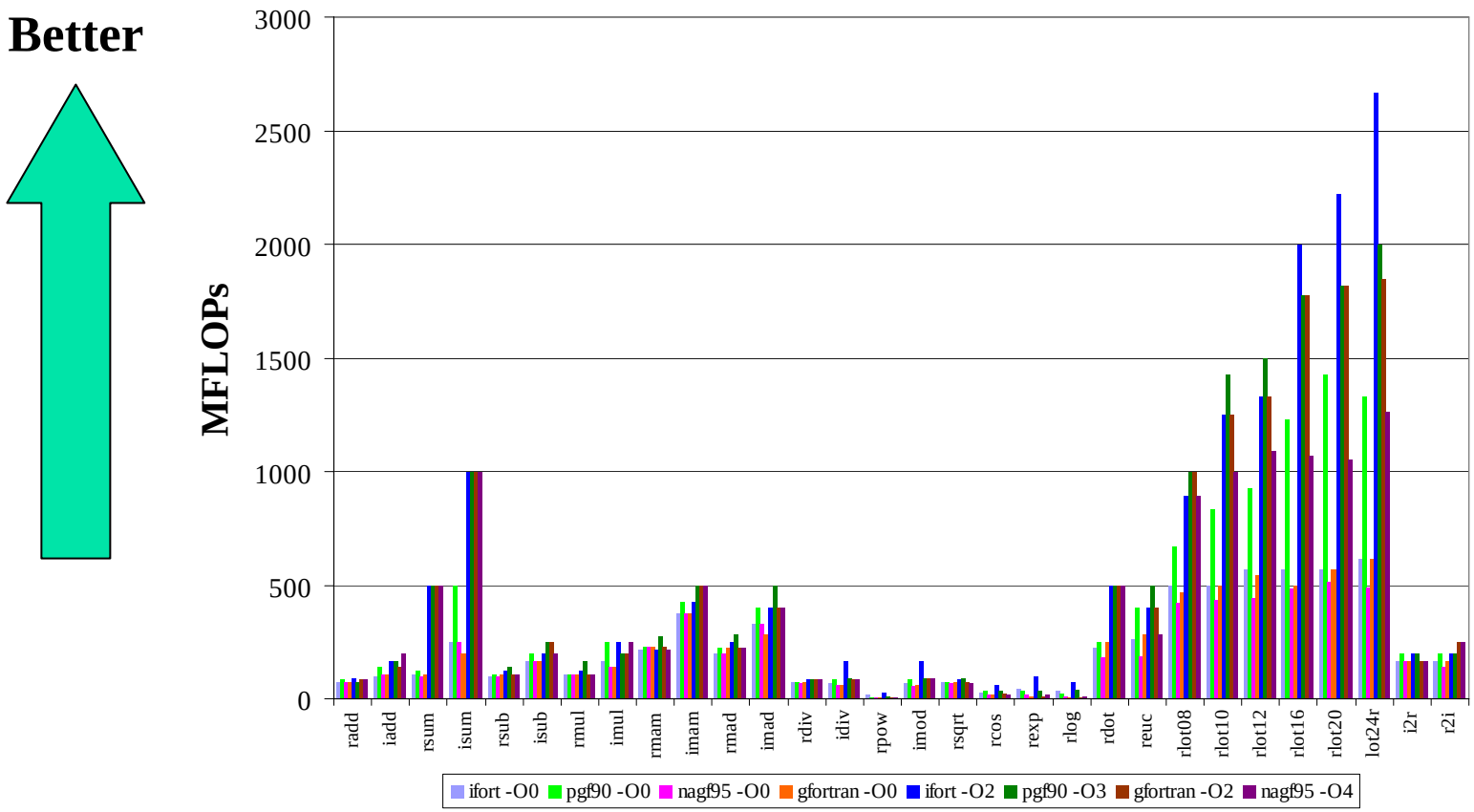
Loop Performance

Performance Characteristics

- Different operations take different amounts of time.
- Different processor types have different performance characteristics, but there are some characteristics that many platforms have in common.
- Different compilers, even on the same hardware, perform differently.
- On some processors, floating point and integer speeds are similar, while on others they differ.

Arithmetic Operation Speeds

Arithmetic Performance on Pentium4 EM64T
(Irwindale 3.2 GHz)



Fast and Slow Operations

- **Fast**: sum, add, subtract, multiply
- **Medium**: divide, mod (that is, remainder)
- **Slow**: transcendental functions (sqrt, sin, exp)
- **Incredibly slow**: power x^y for real x and y

On most platforms, divide, mod and transcendental functions are not pipelined, so a code will run faster if most of it is just adds, subtracts and multiplies.

For example, solving an N x N system of linear equations by LU decomposition uses on the order of N^3 additions and multiplications, but only on the order of N divisions.

What Can Prevent Pipelining?

Certain events make it very hard (maybe even impossible) for compilers to pipeline a loop, such as:

- array elements accessed in **random order**
- loop body **too complicated**
- **`if` statements** inside the loop (on some platforms)
- premature **loop exits**
- function/subroutine **calls**
- **I/O**

How Do They Kill Pipelining?

- **Random access order**: Ordered array access is common, so pipelining hardware and compilers tend to be designed under the assumption that most loops will be ordered. Also, the pipeline will constantly ***stall*** because data will come from main memory, not cache.
- **Complicated loop body**: The compiler gets too overwhelmed and can't figure out how to schedule the instructions.

How Do They Kill Pipelining?

- **`if` statements** in the loop: On some platforms (but not all), the pipelines need to perform exactly the same operations over and over; **`if`** statements make that impossible.

However, many CPUs can now perform ***speculative execution***: both branches of the **`if`** statement are executed while the condition is being evaluated, but only one of the results is retained (the one associated with the condition's value).

Also, many CPUs can now perform ***branch prediction*** to head down the most likely compute path.

How Do They Kill Pipelining?

- **Function/subroutine calls** interrupt the flow of the program even more than `if` statements. They can take execution to a completely different part of the program, and pipelines aren't set up to handle that.
- **Loop exits** are similar. Most compilers can't pipeline loops with premature or unpredictable exits.
- **I/O**: Typically, I/O is handled in subroutines (above). Also, I/O instructions can take control of the program away from the CPU (they can give control to I/O devices).

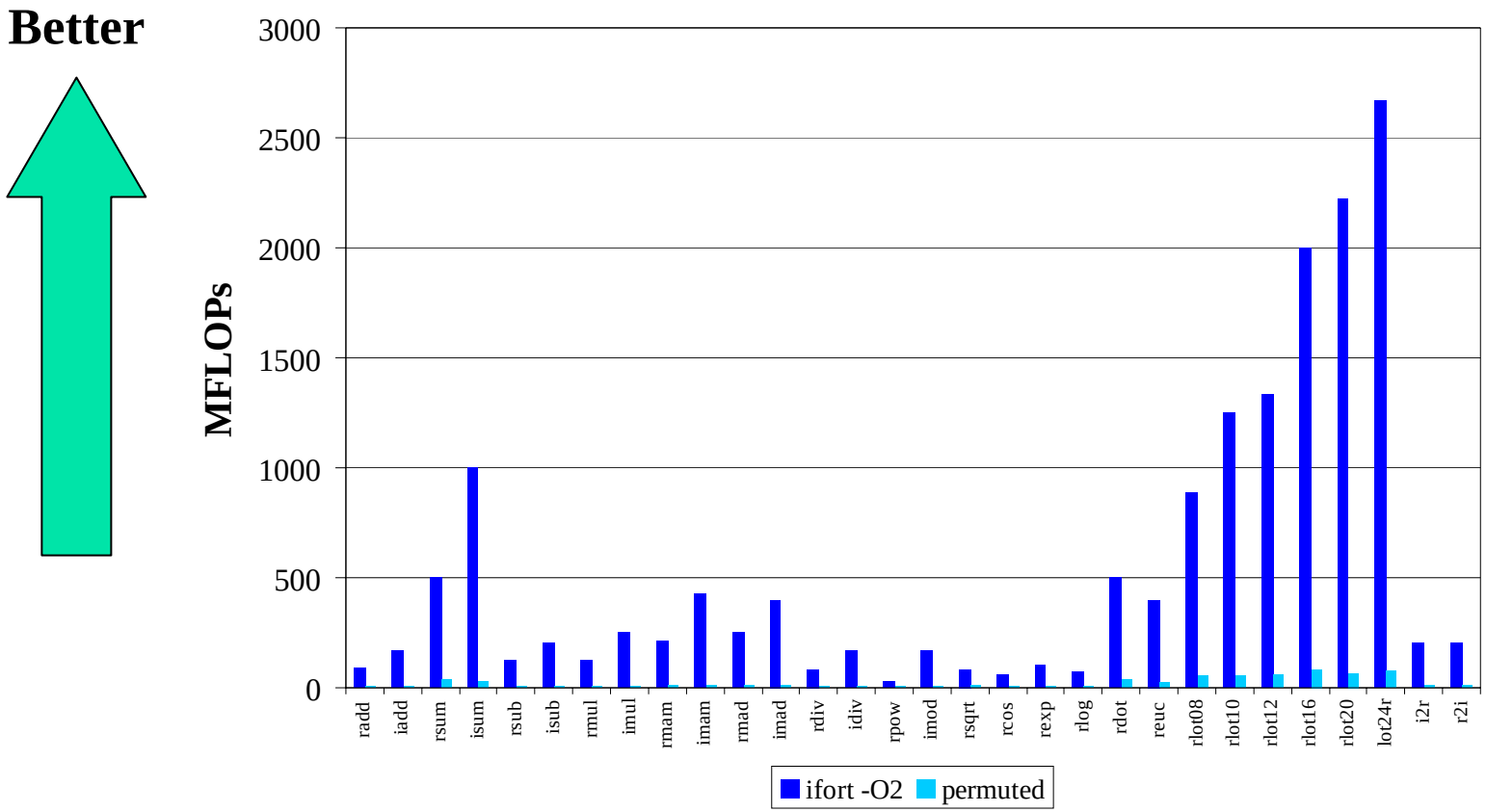
What If No Pipelining?

SLOW!

(on most platforms)

Randomly Permuted Loops

Arithmetic Performance: Ordered vs Random
(Irwindale 3.2 GHz)



Superpipelining

Superpipelining

Superpipelining is a combination of superscalar and pipelining.

So, a superpipeline is a collection of multiple pipelines that can operate simultaneously.

In other words, several different operations can execute simultaneously, and each of these operations can be broken into stages, each of which is filled all the time.

So you can get multiple operations per CPU cycle.

For example, a IBM Power4 can have over 200 different operations "in flight" at the same time.[1]

More Operations At a Time

- If you put more operations into the code for a loop, you can get better performance:
  - more operations can execute at a time (use more pipelines), and
  - you get better register/cache reuse.
- On most platforms, there's a limit to how many operations you can put in a loop to increase performance, but that limit varies among platforms, and can be quite large.

Some Complicated Loops

```
DO index = 1, length
  dst(index) = src1(index) + 5.0 * src2(index)
END DO
```

madd (or FMA): mult then add (2 ops)

```
dot = 0
DO index = 1, length
  dot = dot + src1(index) * src2(index)
END DO
```

dot product (2 ops)

```
DO index = 1, length
  dst(index) = src1(index) * src2(index) + &
 &             src3(index) * src4(index)
END DO
```

from our example (3 ops)

```
DO index = 1, length
  diff12 = src1(index) - src2(index)
  diff34 = src3(index) - src4(index)
  dst(index) = SQRT(diff12 * diff12 + diff34 * diff34)
END DO
```

Euclidean distance (6 ops)

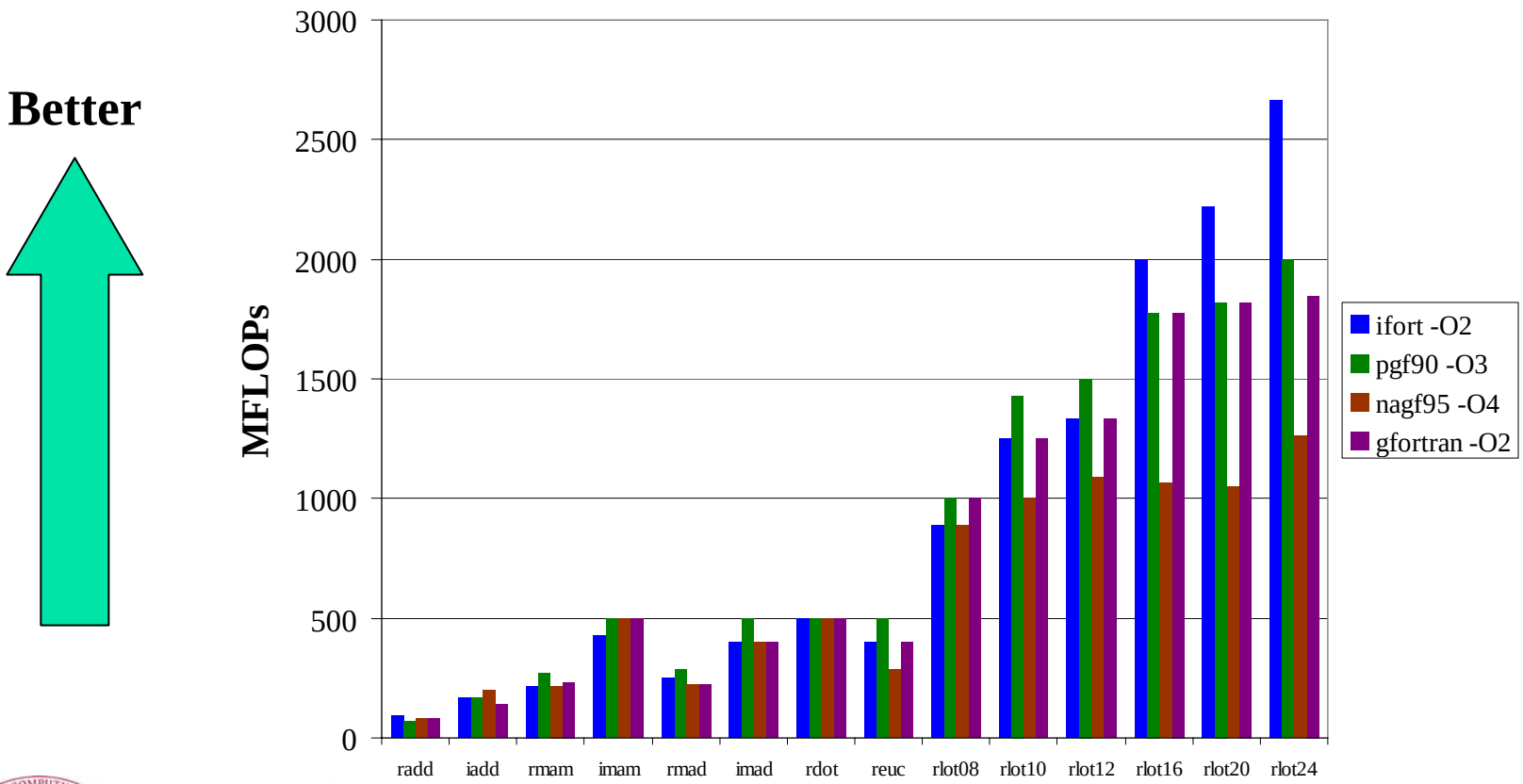
A Very Complicated Loop

```
lot = 0.0
DO index = 1, length
    lot = lot +                              &
 &    src1(index) * src2(index) +            &
 &    src3(index) * src4(index) +            &
 &    (src1(index) + src2(index)) *          &
 &    (src3(index) + src4(index)) *          &
 &    (src1(index) - src2(index)) *          &
 &    (src3(index) - src4(index)) *          &
 &    (src1(index) - src3(index) +           &
 &     src2(index) - src4(index)) *          &
 &    (src1(index) + src3(index) -           &
 &     src2(index) + src4(index)) +          &
 &    (src1(index) * src3(index)) +          &
 &    (src2(index) * src4(index))
END DO
```

24 arithmetic ops per iteration

4 memory/cache loads per iteration

Multiple Ops Per Iteration



Vectors

What Is a Vector?

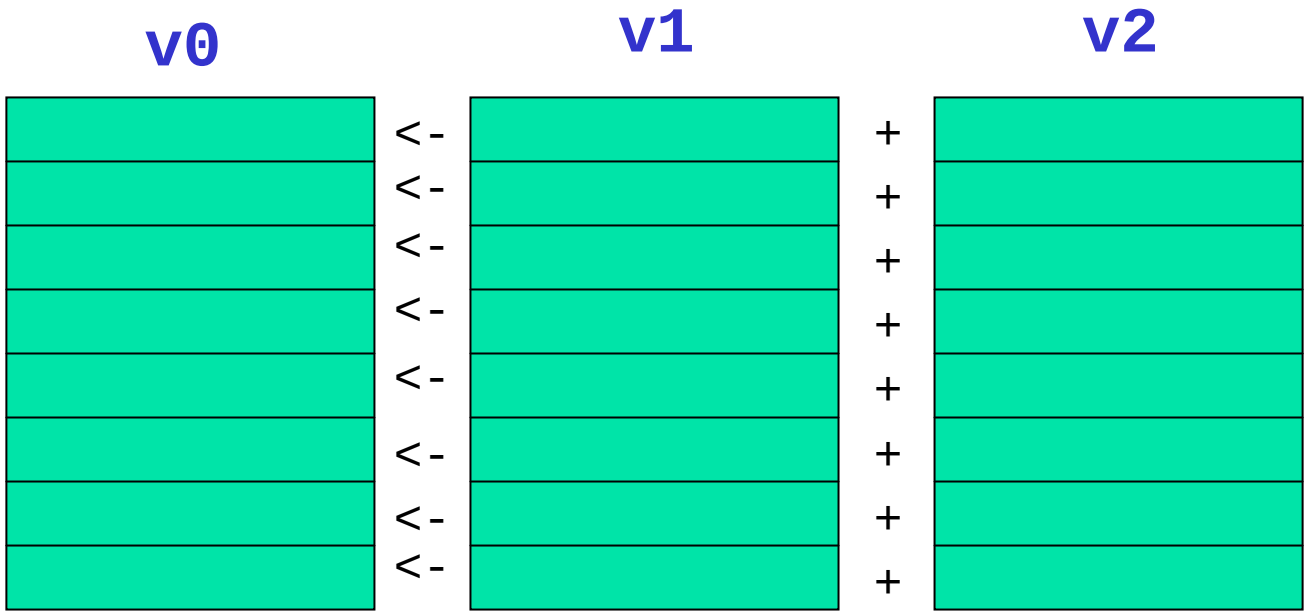
A ***vector*** is a giant register that behaves like a collection of regular registers, except these registers all simultaneously perform the same operation on multiple sets of operands, producing multiple results.

In a sense, vectors are like operation-specific cache.

A ***vector register*** is a register that's actually made up of many individual registers.

A ***vector instruction*** is an instruction that performs the same operation simultaneously on all of the individual registers of a vector register.

Vector Register




```
v0 <- v1 + v2
```

Vectors Are Expensive

Vectors were very popular in the 1980s, because they're very fast, often faster than pipelines.

In the 1990s, though, they weren't very popular. Why?

Well, vectors aren't used by many commercial codes (for example, MS Word). So most chip makers didn't bother with vectors.

So, if you wanted vectors, you had to pay a lot of **extra money** for them.

However, with the Pentium III Intel reintroduced very small vectors (2 operations at a time), for integer operations only. The Pentium4 added floating point vector operations, also of size 2. Now, the Pentium4 EM64T has doubled the vector size to 4.

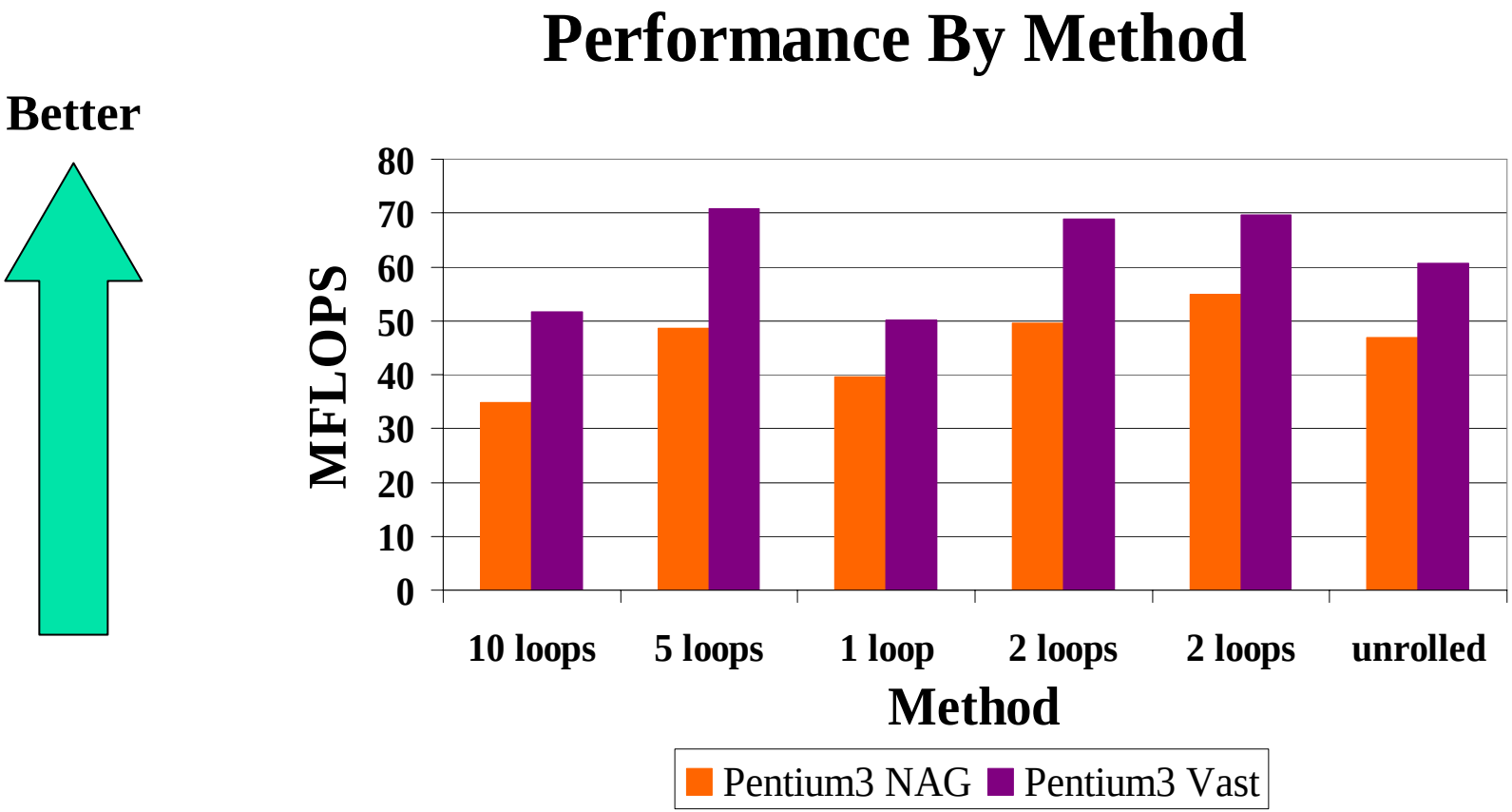
A Real Example

A Real Example[4]

```
DO k=2,nz-1
  DO j=2,ny-1
    DO i=2,nx-1
      tem1(i,j,k) = u(i,j,k,2)*(u(i+1,j,k,2)-u(i-1,j,k,2))*dxinv2
      tem2(i,j,k) = v(i,j,k,2)*(u(i,j+1,k,2)-u(i,j-1,k,2))*dyinv2
      tem3(i,j,k) = w(i,j,k,2)*(u(i,j,k+1,2)-u(i,j,k-1,2))*dzinv2
    END DO
  END DO
END DO
DO k=2,nz-1
  DO j=2,ny-1
    DO i=2,nx-1
      u(i,j,k,3) = u(i,j,k,1) -    &
 &                 dtbig2*(tem1(i,j,k)+tem2(i,j,k)+tem3(i,j,k))
    END DO
  END DO
END DO

. . .
```

Real Example Performance



DON'T PANIC!

Why You Shouldn't Panic

In general, the compiler and the CPU will do most of the heavy lifting for instruction-level parallelism.

BUT:

You need to be aware of ILP, because how your code is structured affects how much ILP the compiler and the CPU can give you.

OK Supercomputing Symposium

Wed Oct 7 2009 @ OU

2003 Keynote:
Peter Freeman
NSF
Computer &
Information
Science &
Engineering
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2004 Keynote:
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2005 Keynote:
Walt Brooks
NASA Advanced
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2006 Keynote:
Dan Atkins
Head of NSF's
Office of
Cyber-
infrastructure

2007 Keynote:
Jay Boisseau
Director
Texas Advanced
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SC09 Summer Workshops

This coming summer, the SC09 Education Program, part of the SC09 (Supercomputing 2009) conference, is planning to hold two weeklong supercomputing-related workshops in Oklahoma, for **FREE** (except you pay your own travel):

- **At OU**: Parallel Programming & Cluster Computing, date to be decided, weeklong, for **FREE**
- **At OSU**: Computational Chemistry (tentative), date to be decided, weeklong, for **FREE**

We'll alert everyone when the details have been ironed out and the registration webpage opens.

Please note that you must apply for a seat, and acceptance **CANNOT** be guaranteed.

To Learn More Supercomputing

http://www.oscer.ou.edu/education.php

Thanks for your attention!

Questions?

References

[1] Steve Behling et al, *The POWER4 Processor Introduction and Tuning Guide*, IBM, 2001.

[2] *Intel® 64 and IA-32 Architectures Optimization Reference Manual*, Order Number: 248966-015 May 2007
http://www.intel.com/design/processor/manuals/248966.pdf

[3] Kevin Dowd and Charles Severance, *High Performance Computing*, 2nd ed. O'Reilly, 1998.

[4] Code courtesy of Dan Weber, 2001.